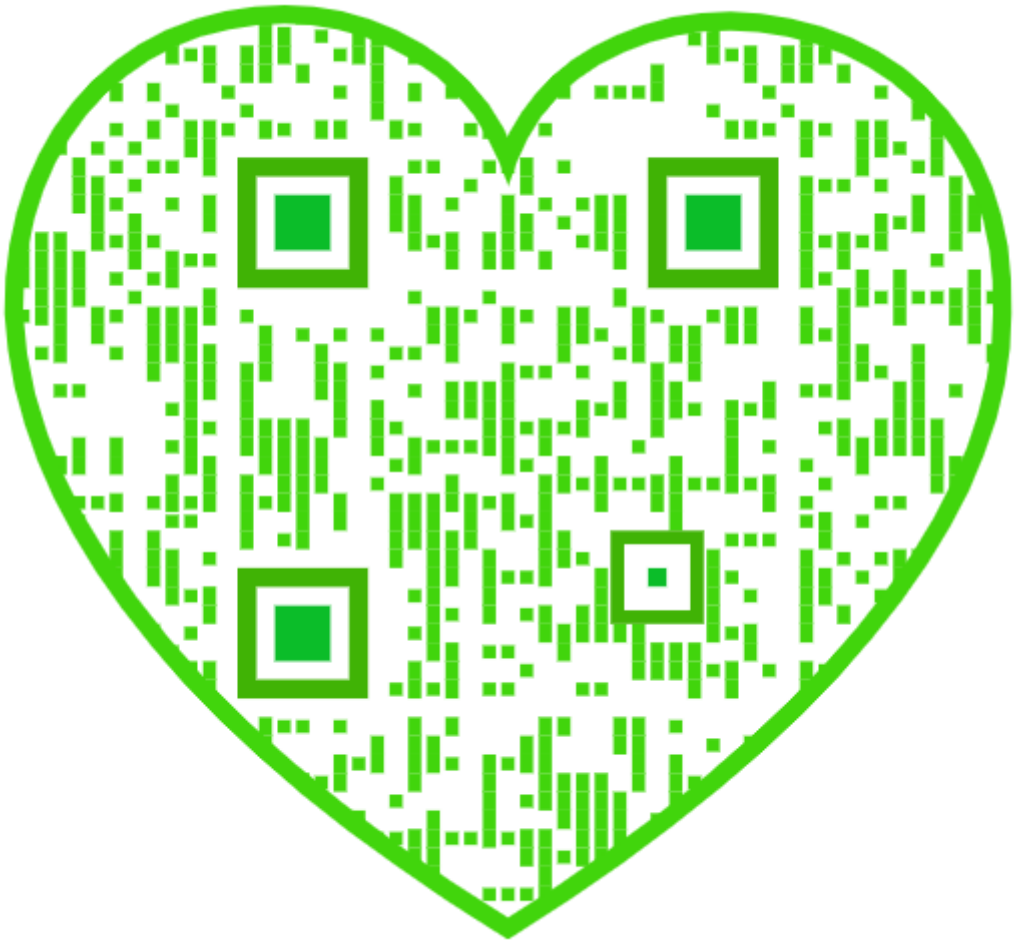

A SHORT COURSE IN THE

TESTING OF

ELECTRICAL MACHINERY

FOR NON-ELECTRICAL STUDENTS

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PREFACE

In presenting these brief notes the authors feel that an explanation of their object is necessary. At Columbia University practically all of the engineering students are required to take courses in the electrical laboratories, testing both direct-current and alternating-current machinery. Students in Mining, Mechanical, Metallurgical, Chemical, Civil Engineering, etc., do not have those courses in the theory of electrical machinery, which are really necessary for a proper comprehension of the machines with which they work in the laboratory; it is unreasonable to expect them to consult various text-books to prepare themselves on the theory involved in the tests, and it is with the intention of filling the needs of these men that the notes have been compiled.

Before giving specific directions regarding the test to be performed, a brief analysis of the characteristics of the machine is attempted; in so far as is possible in such a limited space the reasons for the behavior of the machine are given. It is, of course, realized that a complete analysis of the different types of machines is impossible and it is questionable whether a complete analysis would serve the purpose. It has been the intention of the writers to present the subject-matter in such a manner that the student not well versed in electrical theory can get the most out of it in the short time allotted to the electrical courses.

In some of the tests, methods are described which may not be strictly according to the standard practice; if a gain in simplicity and ease of performance is to be obtained by a sacrifice in accuracy of the test of a few tenths of a per cent, it is thought justifiable to use the simpler method of testing.

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While the notes are being put into printed form specifically for our use, they may possibly be found of use in other schools where the conditions are similar to those at Columbia.

The authors wish to express their indebtedness to Professor Geo. F. Sever, who first developed the electrical laboratory work for the non-electrical students at Columbia, and whose original schedule of experiments served as a guide in arranging this work; also to Mr. F. L. Mason, who has rendered valuable assistance in the preparation of the book.

J. H. M.
F. W. H.

COLUMBIA UNIVERSITY,
September, 1911.

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TESTING OF ELECTRICAL MACHINERY

DIRECT CURRENT TESTS

EXPERIMENT I

"Fall of Potential" along a Conductor Carrying Current. (*a*) If an electromotive force is impressed upon a circuit, as for instance a wire, a current of electricity will flow along it. The relation between the current, resistance and difference of potential between any two points on the conductor is given by Ohm's law, which expresses the equality of the impressed force and the reacting force. It is found that the reacting force varies directly with the current and this fact may be expressed by the equation,

$$E=IR$$

from which we obtain,

$$I=E/R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I=current flowing in amperes;

E=difference of potential in volts, between the two points considered;

R=resistance of the conductor in ohms, between the two points considered.

It is a fact that all conductors offer more or less resistance to the flow of an electric current and experiment shows that for any particular conductor, the resistance varies directly as its length and inversely as its area of cross-section. This may be expressed in the form of an equation as follows:

$$R=K\frac{l}{a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R = resistance of conductor in ohms;
l = length of conductor in feet;
a = area in circular mils;
K = resistance per mil foot of the material used; i.e., the resistance of a conductor one foot long and having a cross-sectional area of one circular mil.

If the value of R as given in Eq. (2) is substituted in Eq. (1) we have

$$I = \frac{E}{K\frac{l}{a}} = \frac{Ea}{Kl}, \quad \ldots\ldots\ldots \quad (3)$$

and

$$E = IK\frac{l}{a}. \quad \ldots\ldots\ldots \quad (4)$$

This states that when the other factors remain constant, the "drop" in potential varies directly with the current and the length of the conductor and inversely with the area of cross-section. The drop also varies with the material.

It is also evident if Eq. (1) is written in the form

$$R = \frac{E}{I}, \quad \ldots\ldots\ldots \quad (5)$$

that the resistance of a conductor is readily determined if the current flowing through it and the drop in potential across it are known.

The apparatus to be used in verifying the laws stated above should consist of a board, upon which are mounted and connected in series, equal lengths of wire (preferably 48 inches) of copper, aluminum, iron and German silver, all of equal cross-section. Connect the wire board as shown in Fig. 1. Determine the safe current-carrying capacity of the German silver wire and be sure that the variable resistance, lamp board, and ammeter can safely carry this current. The voltmeter should have a capacity on one range of about 15 volts with another range lower than this.

With only one lamp in the lamp board and all the resistance in the variable rheostat cut out, close the switch and note if the ammeter deflects in the proper way. If not, open the switch and reverse the ammeter connections. Then cut in enough additional lamps in the board, so that the current will be slightly greater than $\frac{1}{3}$ of the safe current value for the German silver wire. Insert enough resistance in the variable rheostat so that exactly $\frac{1}{3}$ of the maximum permissible current is flowing. Using the highest range upon the voltmeter, place one lead to one end of the copper wire and tap the other voltmeter lead on the 6-inch mark upon the wire. If the throw of the needle is in the proper direction, the free lead may be fastened to the wire; if not, the

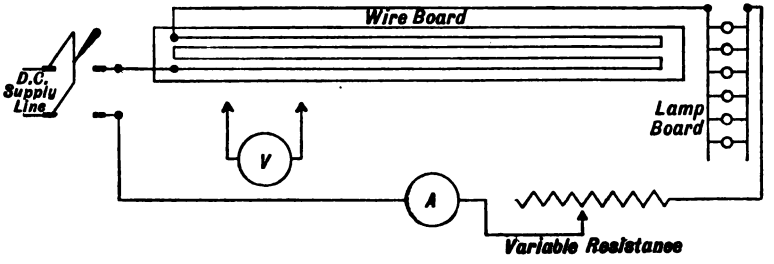

FIG. 1

leads are to be reversed *at the wire. Never reverse or disconnect voltmeter leads at the meter, while the other ends of the leads are attached to live terminals; a short circuit is likely to result. Always reverse or disconnect the leads at the point where they are attached to the live terminals.* Great precaution must also be taken with low-reading voltmeters, as they are easily burnt out. To determine an unknown low voltage, a range somewhat above the line voltage should first be tried and a reading taken; lower ranges can then be tried until the proper one has been determined.

Having determined the proper range upon the voltmeter and adjusted the variable resistance so that the current flowing is exactly the desired value, read the voltmeter. Move the voltmeter leads to include 12 inches of wire and repeat. Continue this operation until the drop over the entire length of copper wire

has been determined and then do the same with the remaining three wires. Raise the current to $\frac{3}{4}$ and finally to the full value of the current-carrying capacity of the system and measure the fall in potential along each wire. Record readings in neatly tabulated form and calculate the resistance of each wire for each value of current. Calculate how much power was used in each wire for each value of current. Determine the average value of the resistance per mil foot for each material and with the value obtained calculate the resistance of 1000 feet of No. 10 B. & S. wire, obtaining the diameter of No. 10 wire from a wire table.

(*b*) Determine the hot and cold resistance, total watts and watts per candle power for a 16- and a 32-candle power carbon lamp for a 20-candle power tantalum lamp, and for a 20-candle power tungsten lamp

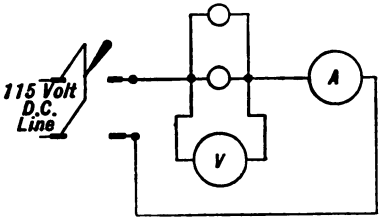

FIG. 2

Carbon has a negative temperature resistance coefficient and metals have a positive coefficient. Accordingly, it will be found that the cold resistance of a carbon lamp is about double its hot resistance, while for the metal filament lamps the hot resistance is many times the cold resistance.

To determine the hot resistance of the lamps, connect them as in Fig. 2, to a source of E.M.F. of about 115 volts, in series with an ammeter of about 2 amperes range. No series resistance is required, as the lamps themselves are of high enough resistance to withstand the full line voltage. Using a voltmeter with a range of 150 volts, determine the voltage across the lamp burning. Remove the voltmeter and read the ammeter. Do not read the ammeter with the voltmeter connected across the lamp, as the current indicated by the ammeter will be the sum of the lamp and voltmeter currents and not the lamp current alone. Take a set of readings for each lamp and record them in a log as shown in Table I.

TABLE I

Type of Lamp.	Candle Power.	Volts.	Amperes.	Watts.	Watts per Candle Power.	Resistance.	
						Hot by Drop in Potential	Cold by Wheatstone Bridge.

Determine the cold resistance of the lamps by means of a Wheatstone bridge.

In calculating watts per candle power, assume that the lamp gives it rated candle power at the voltage used. The watts used per candle power serves as a measure of the efficiency of the lamp, in that a lamp is usually rated as using so many watts per candle power; a lamp using 1.2 watts per candle power is evidently more efficient as a light producer than one using 3 watts per candle power.

Curves. (*a*) Plot the results obtained for each wire on one sheet of cross-section paper, plotting voltmeter readings as ordinates and lengths as abscissæ.

Conclusions. How does drop in potential along a wire vary? Why are the curves straight lines? What might cause them to be slightly concave upward? Which metal is the better conductor and which best adapted for use as resistance wire in rheostats and why? Why did the wires expand during the experiment? How do the values for resistance per mil foot and for the resistance for 1000 feet of No. 10 B. & S. wire compare with values obtained from wire tables? How do you explain the difference in the hot and cold resistances of carbon, tungsten, and tantalum lamps? How do the efficiencies of the different types of lamps compare? Explain briefly the principle and operation of the Wheatstone bridge.

EXPERIMENT II

Measurement of Armature Circuit and Shunt Field Resistances. (*a*) The resistance of an armature circuit is made up of the resistance of the conductors upon the armature, the brushes, the brush contacts, and the cables leading from the brushes to the machine terminals. In a well-designed motor or generator, the armature circuit resistance is made as low as is consistent with the size of the machine, in order to cut down the amount of energy dissipated as heat. The rate of production of heat in the armature is given by the formula, watts $= I^2R$. A motor or a generator is designed to carry a certain maximum value of armature current and this then fixes the value for I, so that to keep the amount of heat generated in the armature low, the resistance must be made as small as is commercially practical.

The armature conductors being always of copper, their resistance will be independent of the current except for heating. The same applies to the machine leads, the resistance of which is very small. The brush contact resistance is the resistance of the surface contact between the carbon brushes and the commutator. This resistance is quite appreciable and decreases with increase of current strength; it decreases as the mechanical pressure between the brush and the commutator increases. The resistance of the brushes themselves is insignificant.

Determine the full load armature current of the machine, whose armature circuit resistance is to be measured, from its name-plate data; connect the armature in series with an ammeter, lamp board and a variable resistance, all of suitable current carrying capacity, as shown in Fig. 3. Close the line switch and adjust the current to $\frac{1}{8}$ of the full-load value.

Using first the highest range of the voltmeter, attach the leads

to the two terminals of the machine (corresponding to position 1 in Fig. 3) and read the meter. If the reading is within the next lower range, detach the leads *from the machine terminals* and shift to the lower range. When a suitable range has been found upon the voltmeter, so that the deflection is a large one, read simultaneously ammeter and voltmeter and record the

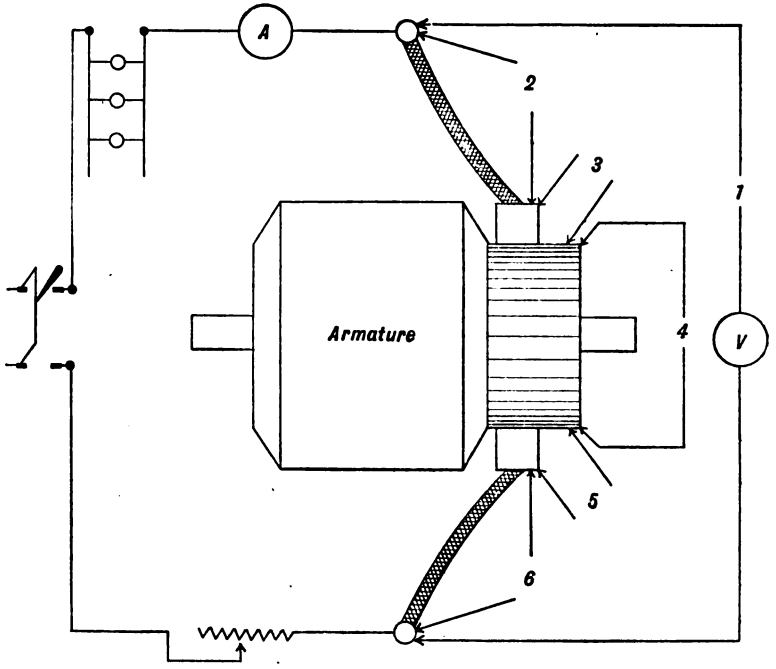

FIG. 3

readings in a log as in Table II. From the readings taken with the voltmeter leads attached as in position 1, the resistance of the entire armature circuit can be calculated. Leave one of the voltmeter leads upon one of the machine terminals and starting from the one employed, trace the cable which leads to the machine. Find the brush to which this cable is attached and connect the other voltmeter lead to some convenient place upon it (position 2). After having adjusted the current to its

proper value read the voltmeter. From this reading the resistance of one of the machine leads can be determined. Then shift the voltmeter leads successively to positions 3, 4, 5, and 6, reading the meters as before. These readings should be taken as rapidly as possible, so that the armature shall not heat up too much. Repeat this operation, using 2/3 and full-load armature current, respectively. The sum of readings 2, 3, 4, 5, and 6 entered in column 7 of the log should equal the values in column 1, the measured drop of the entire circuit. Calculate the values of resistance and tabulate in a form similar to Table II.

TABLE II

Amperes.	Volts.							Difference between 1 and 7.
	1	2	3	4	5	6	7	
	Across entire Armature. Circuit.	Across ends of one Machine Lead.	Across one Brush Contact	Across Armature.	Across second Brush Contact.	Across ends of second Machine Lead.	Sum of Drops 2, 3, 4, 5 and 6.	

Caution. Great caution must be exercised in obtaining the voltage across the various parts. In order to obtain large readings upon the voltmeter it will be necessary to shift from one low reading range to another, *so that it must constantly be borne in mind that a voltmeter needle is easily bent or the instrument entirely burnt out, if a high potential is applied to a low range.*

To determine brush drop, connect one voltmeter lead to the place upon the brush or brush stud used in determining the drop across the machine leads and hold the other lead on the commutator bar directly under the brush; do not touch the terminals of the voltmeter leads against the brushes. When determining the drop across the armature winding hold the voltmeter leads to commutator segments which are under brushes of opposite polarity, again being careful not to touch

the terminal on the commutator, against the brush while reading the voltmeter.

Do not permit the armature to rotate, as a counter electromotive force is generated which decreases the current and causes an apparent increase in the resistance of the armature.

Caution. When it becomes necessary to open the armature circuit be sure that the voltmeter leads have been entirely removed before the switch is opened. A large amount of magnetic energy is stored up in the armature while current is flowing and when the circuit is opened this energy must be dissipated. If a voltmeter is connected across the armature terminals a high protential will be applied to it, due to the self-induction of the armature. This is quite likely to bend the needle of the voltmeter and may even burn it out. Therefore never suddenly open a circuit containing much self-induction while a voltmeter is connected across any part of it.

(*b*) The function of a shunt field is to provide a certain number of ampere-turns for the magnetic circuit and this can be accomplished either by using a small number of turns carrying a comparatively large current, or a large number of turns with a small current flowing through them. Generally a large number of turns is preferred, as this gives a high field resistance and a comparatively small consumption of energy in the field coils. A shunt field is always so built that it may be safely placed across a line of the same voltage for which the machine was designed.

It must be remembered that a shunt field possesses considerable self-induction, so that when a difference of potential is applied to it, the current does not immediately assume its final value, but builds up slowly. This is due to the counter-electromotive force of self-induction which is set up while the current is increasing. This "building up" of the current can be seen very nicely by watching the ammeter as the line switch is closed.

Connect the shunt field whose resistance is to be measured, in series with an ammeter of proper capacity, to a source of E.M.F. equal to that for which it was designed; the connections to be as

shown in Fig. 4. Close the line switch and using a voltmeter of the proper range, determine the difference of potential between the ends of the field. Remove the voltmeter and determine the current value. Then open the circuit slowly, drawing out the arc; be sure the voltmeter is disconnected.

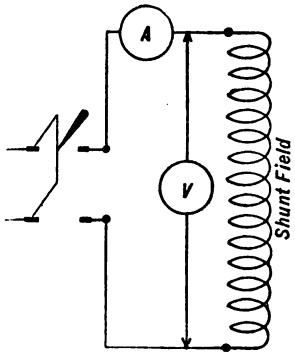

FIG. 4

Also determine the resistance of the field, using a Wheatstone bridge.

Conclusions. Explain why armature resistances are made low and shunt field resistances high. How do the resistances of the various parts of the armature circuit vary with current? Why is it unsatisfactory to determine the armature circuit resistance with the ordinary Wheatstone bridge. Why should an inductive circuit never be opened if a voltmeter is connected across any portion of it?

EXPERIMENT III

The Shunt Generator. (*a*) Preliminary Work with a Generator; (*b*) Magnetization Curve of a Shunt Generator; (*c*) External Characteristic of the Shunt Generator.

(*a*) The student should first familiarize himself with the component parts of the machine assigned and look over their construction and relation to one another, in order to get a general idea of the generator. The following table of data and dimensions is then to be completely filled out.

General

Type of generator.....shunt or compound	Current.....
	K. W. Output.....
Rated voltage.....	Speed .....

Armature

Type.....ring or drum	Active length .....
Number of sections.....	Circumference in feet.....
Conductors per section.....	Peripheral speed in ft. per min.
Total number of conductors. ..	

Commutator

Active length.	Average voltage between bars .
Total length.....	Width of bars. . .....
Circumference in feet.....	Width of insulation .....
Peripheral speed in ft. per min..	Kind of brushes.....
Total number of bars.....	Brush area per set.....
Number of bars between brushes of opposite polarity.....	Current density in brushes, in amperes per sq. in .....

Fields

Number of main poles.....	Pole width .....
Number of commutating poles..	Pole length .....
Pole arc in degrees	Width of pole shoe .....

A shunt generator can readily be distinguished from a compound generator by examination of the field coils. A shunt field is wound of relatively small wire and there will be only two terminals on each field spool. A compound generator has a series winding of large wire or copper strip in addition to its shunt winding, so that each field spool will have two large additional taps for the series field current. The remainder of the general data can be determined from the name plate.

The type of armature can generally be determined by examining the back connections. A ring winding is one in which the wire is wound around an annular ring, so that there are conductors both on the outside and the inside of the ring. In a drum winding all of the conductors are upon the other periphery. An armature section can be defined as that part of the winding, which starts at one commutator bar and ends at another. The number of sections upon the armature can sometimes be determined from the back connections, the number of slots and the connections to the commutator. The number of sections will never be less than the number of commutator bars and will generally be some multiple or sub-multiple of the number of slots. If the manufacturer's data is at hand the number of sections can be taken therefrom and then verified. The number of conductors per section will probably have to be taken from the manufacturer's winding data. The active length of the armature will be equal to the width of the pole shoe (i.e., the length of the pole shoe parallel to the shaft) plus twice the length of the air gap.

The active length of the commutator can be regarded as the distance occupied by the brushes plus the spaces between them, measured parallel to the shaft. A commutator is always made longer than this to permit clearance and end play. In determining the current density of the brushes, remember that brush sets always operate in pairs. In a bi-polar generator each brush set carries the total current of the machine. The pole width is the distance parallel to the shaft. The pole length is measured out radially.

Before a generator is to be operated, certain conditions should

be noted, particularly in a new machine or one which has not been run for some time. The things to be noted are as follows:

(1) Amount of oil in the bearings or cups. With self-oiling bearings it is advisible to note whether the oil rings turn easily and dip into oil. This can be readily seen by turning the rings over by hand.

(2) Condition of the brushes and commutator. The commutator should be clean and fairly bright. If it is not, it can be polished while revolving with fine sandpaper on a flat wooden block or a polishing stone. Do not use emery. The brushes should make good contact over their whole bearing surface.

(3) Brush Tension. This should be from one and a half to two pounds per square inch of contact surface.

(4) The armature should turn easily, either by hand in the case of a small machine or by means of a lever in a large one. If the armature turns hard, either the bearings are in bad condition or the alignment of the shaft in the bearings is not good.

(*b*) A generator is a device which transforms mechanical energy into electrical energy. Its operation is based upon the fact that when a conductor is moved in a magnetic field so as to cut lines of force, an E.M.F. is induced in it. In a generator a number of copper wires or bars are mounted upon a cylindrical iron core, and this armature, when rotated in a magnetic field, generates a voltage. The armature inductors are properly interconnected, so as to add their individual E.M.Fs. and are also connected to the commutator which rectifies the alternating voltages induced in them.

The fundamental equation of the generator can be written as follows:

$$E_g = \frac{\phi \times N \times n \times 2p}{10^8 \times 60 \times c}, \quad \ldots\ldots\ldots \quad (6)$$

where E_g is the voltage generated by the armature;

ϕ = the flux per pole in lines of force;

N = R.P.M.

n = total number of inductors upon the armature;

p = number of pairs of poles;

c = number of circuits in parallel upon the armature.

The derivation follows if we remember that when an inductor cuts 10^8 lines of force per second, an E.M.F. of one volt is induced between its ends.

The magnetic field of a generator is produced by the ampere-turns of the field windings according to the formula,

$$\text{Magnetomotive force} = .4\pi \times \text{ampere-turns},$$

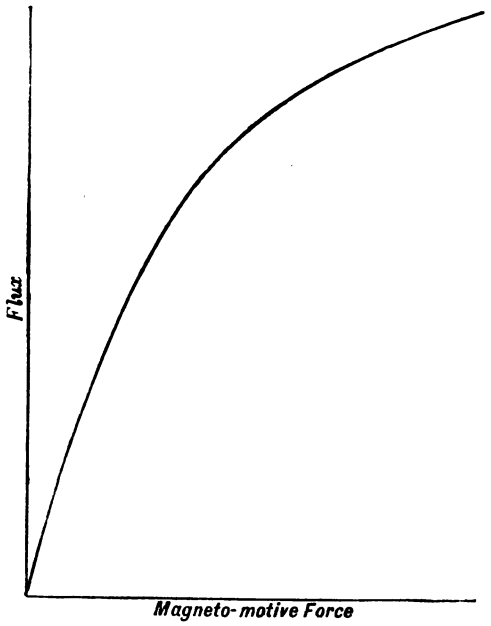

FIG. 5

and this magnetomotive force acting over the reluctance of the magnetic circuit produces a flux according to the relation

$$\text{flux} = \frac{\text{M.M.F.}}{\text{reluctance}}.$$

The curve showing the relation between flux and M.M.F. is shown in Fig. 5; it is seen that at low values of M.M.F. a small increase in M.M.F. produces quite a large increase in flux. As the field becomes stronger the flux curve begins to bend over

more and more, so that for large additions of M.M.F. there is only a small change in the flux; when in this state the iron is said to be saturated.

In the case of a generator, in determining the magnetization or saturation curve, we make use of the fact that magnetomotive force is directly proportional to the field amperes, since the number of field turns is constant. Furthermore, in Eq. (6), if the speed is a constant quantity, the generated voltage will be a measure of the flux, all the other quantities being constants. The magnetization curve of the generator may then be plotted between generated volts and field current.

It is preferable that the field of the generator be separately excited, and a very convenient method for doing this is the so-called potentiometer method. This consists of a rheostat, with a sliding contact, placed across the D.C. line as shown in Fig. 6. By adjusting the slider *S*, any desired voltage can be had across the leads *PQ*, from zero up to the value of the supply voltage. If the leads *PQ* are applied to a shunt field it is possible to vary the field current gradually from zero to its maximum value.

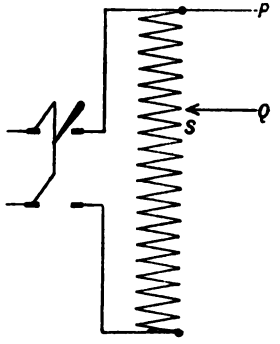

FIG. 6

Excite the field of the machine to be used, from a source of E.M.F. greater than the rated voltage of the machine, using potentiometer connection as is shown in Fig. 7. For a 115-volt generator, excite the field from a 230-volt line, etc. With the line switch open, rotate the machine at its rated speed by means of a prime mover whose speed can be held constant; place a voltmeter of suitable range across the brushes of the machine. With no current flowing through the field of the machine, it will be found that a small voltage is generated. This is due to a small amount of residual magnetism retained from a former excitation.

Now close the line switch and send a small current through the field of the generator, making sure that its direction is such

as to increase the voltmeter reading. Then keeping the speed of the machine constant, take about 10 readings, raising the voltage 50 per cent above its rated value. Then decrease the value of current in similar steps until it is again zero. It will be found that the descending curve is higher than the ascending, or in other words that the same field current gives a higher generated voltage descending than ascending. This is due to hysteresis or the retentiveness of the magnetic path. The curve obtained is shown in Fig. 8.

Caution. Because of hysteresis, great care must be taken when on the ascending curve, to always bring the magnetizing

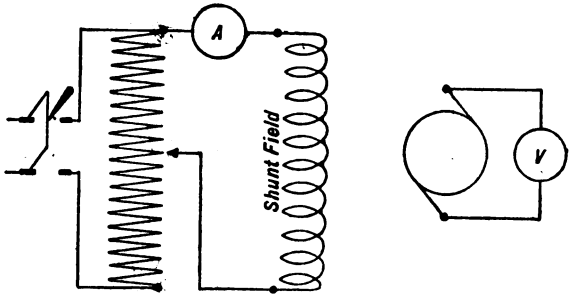

FIG. 7

current up to the value at which a reading is to be taken. If the desired value should be exceeded and the current subsequently reduced to it, the readings obtained would lie on the descending curve. In case the desired value of field current is exceeded, adjust the current to the proper value, open the field circuit slowly for an instant and then let the current build up again. The opposite applies to the descending curve.

(*c*) A shunt generator, as its name implies, is one in which the field is shunted or connected directly across the armature. Thus a part of the armature current is diverted from the external circuit and sent through the field. As has been seen before, the resistance of the shunt field is made high in order that only a small percentage of the current output of the generator may be used in this part of the machine: the power expended in the field

circuit is lost as heat, and being a constant loss, every effort is made to reduce it to as low a value as is practically possible.

It was mentioned in part (*a*) that even with no field current a small flux due to residual magnetism traverses the armature, which the conductors cut as they rotate. If the shunt field is

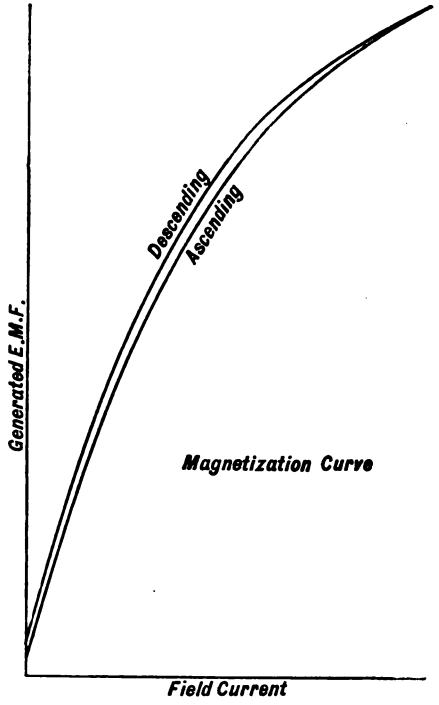

FIG. 8

connected across the brushes, the small E.M.F. generated by the armature, causes a small current to flow through the field winding, which strengthens the residual field. This increased field induces a higher E.M.F. in the armature, which in turn causes more field current, so there results a stronger field, and so on until the excitation reaches its proper value. This process is called "building up." It is quite evident that if there is no residual magnetism present, as may sometimes occur, that the machine

cannot "build up" any voltage. In this case the field must be excited by some exterior source of power. Another possibility of trouble is reversed field connections, so that whatever voltage is generated due to residual magnetism, causes current to flow through the field coils in such a direction as to weaken the residual magnetism instead of strengthening it. To test for this, open the field and determine the voltage due to residual magnetism. Then close the field and notice if the voltmeter drops toward zero. In this case reverse the field connections. If there is a short circuit in the external circuit the machine will not build up, nor can it if the field connections are open. If the residual magnetism is weak, increasing the pressure of the brushes will often start a machine to build up its voltage.

The external characteristic of a shunt generator is the curve which shows the relation between terminal voltage and external current. If we have a generator rotating at rated speed and allow it a definite field current, it will generate a certain E.M.F. If the external circuit is closed through a resistance, a current will flow through the armature and the external circuit, depending upon the value of the voltage generated and the resistance of the external circuit. It has been seen before that when current flows through an armature, there is a drop of potential, so that some of the voltage induced by the generator will be used up in the armature and the remainder in the load circuit. We may write this in the form of an equation as follows:

$$E_g = E_t + I_a R_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where E_g = voltage generated by the machine;
E_t = terminal voltage;
I_a = current flowing through the armature;
R_a = resistance of the armature circuit.

As more and more load is placed upon the machine the terminal voltage is decreased due to the increased IR drop in the armature. In addition to this there is also armature reaction to be taken into account, the effect of this being generally to weaken

the field and thus reduce the generated voltage. Furthermore, the shunt field, being also connected across the terminals of the machine, receives less and less current due to the terminal voltage decreasing which still further weakens the flux. It should be carefully noted that the decrease in shunt field current, is a result of

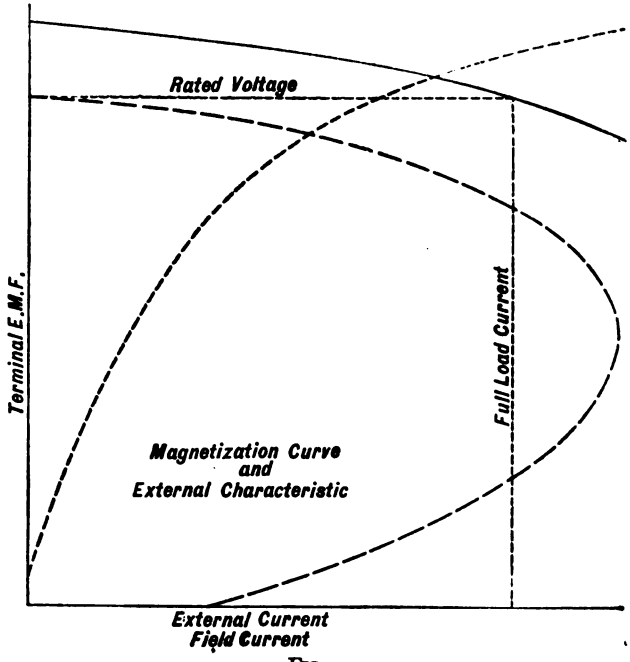

FIG. 9

increased armature IR drop and armature reaction, for if the terminal voltage had not fallen due to these two phenomena, the shunt field current would have remained constant.

It is evident from the above that the greater the armature resistance the larger will be the armature IR drop and the more the voltage will fall off with the load; armature reaction should also be kept as small as possible.

As the external resistance is decreased more and more, a point is reached where the external current no longer increases but actually decreases. If the external resistance is still further

decreased the current continues to decrease until when dead short circuit is reached there is only a small current flowing. The E.M.F. generated in the armature under such short circuit is due to the residual flux and it is nearly all used up as IR drop in the armature, the terminal voltage being practically zero. The reasons for the curve bending back on itself can be seen by comparing the curve with the magnetization curve as in Fig. 9. The machine with no external current flowing is operating at a point upon the saturated portion of the curve. When the field weakens a little due to the addition of load as stated before, the point of operation is then lower down, but as the field is still somewhat saturated the change in flux is not great. However, when the machine drops down to the straight portion of the magnetization curve, where a small change in field current causes a large decrease in flux, then the generated and terminal voltages fall away quite rapidly and the external current decreases.

To determine the external characteristic, the machine should be so adjusted that when it is supplying full load current its terminal voltage is at its rated value. However, if this were done, the armature current would reach an excessive value before it would turn back on itself, and it is therefore advisable to obtain a curve of this peculiar form by starting with the machine at its *rated* votage at *no* load. Such external characteristic is illustrated by the broken curve in Fig. 9.

According to modern practice nearly all large generators and most small ones are equipped with commutating poles, by whose aid, sparking at the commutator and the necessity for brush shifting are done away with. The presence of commutating poles may to some extent alter the shape of the external characteristic so that it is preferable to use a generator not so equipped to determine this curve.

Connect the machine as indicated in Fig. 10, using an ammeter of the proper range in the field and one in the external circuit whose range will equal twice the rated full load current of the machine.

Adjust the machine to normal voltage at full load and rated

speed and shift the brushes to the position giving best commutation. Without altering the field rheostat remove the load, and after making certain that the speed is still at its rated value read the voltmeter. The per cent rise in voltage determines the regulation of the machine, as the regulation of a generator is given by the equation

$$\text{Regulation} = \frac{\text{no load voltage} - \text{full load voltage}}{\text{full load voltage}}.$$

Shift the brushes to give no sparking and then lower the voltage of the generator at no load to its rated value, and after noting that the speed is at its proper value read the meters. (If the generator used is equipped with commutating poles, it will not be necessary to shift the brushes if they were properly set to start.) Then keeping the speed constant and without altering the field resistance or shifting brushes, continue

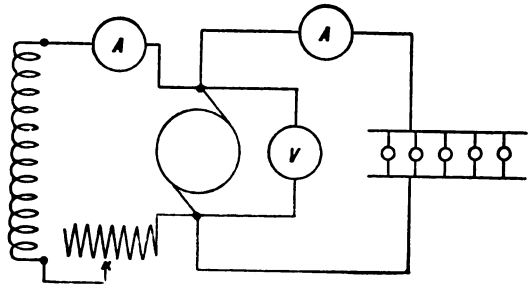

FIG. 10

adding load in the form of lamps or water rheostats in equal increments, each time taking a reading upon all the meters. Continue this until the voltage is low enough for the machine to be completely short circuited. Record readings and calculate the armature *IR* drop as indicated in Table III.

TABLE III

Speed.	Terminal E.M.F.	Armature Current.	Field Current.	Armature *IR* Drop.

After this test (using a generator not provided with commutating poles), bring the machine to rated voltage at rated speed with no load upon it, adjusting the brushes to give best commutation. Then add full load and shift the brushes forward until the point of best commutation is again reached; watch the voltmeter very carefully while this is being done. Then shift the brushes further forward and then backward beyond the no-load neutral point, again noting the voltmeter and also sparking.

Finally measure the armature resistance for several values of current from zero to the maximum value used. Plot a curve between armature current and armature resistance and use this curve in calculating IR drop in table.

Curves. Plot the magnetization curve of the machine from the results obtained; plot the external characteristic on the same sheet, using same scale for E.M.F.

Conclusions. What does the magnetization curve show? Why are the ascending and descending curves not coincident? What purpose does residual magnetism serve? For what reasons might a generator refuse to build up and what action is necessary to remedy the trouble? Explain why the terminal voltage of a generator tends to fall with increase of load. Why should the armature resistance of a generator be made low? Explain briefly the principles upon which armature reaction and sparking depend. Why does the presence of commutating poles eliminate sparking and the necessity of shifting brushes.

EXPERIMENT IV

The Compound Generator. (*a*) Armature Characteristic of a Shunt Generator. (*b*) External Characteristic of a Compound Generator. (*c*) Effect of Operating a Compound Generator at Speeds Higher or Lower than Rated Value.

(*a*) We have seen that when a shunt generator is loaded, the terminal voltage falls if no attempt is made to regulate it, the decrease being due to IR drop in the armature and armature reaction. As a result of these, the shunt field current was reduced, causing a still further decrease in voltage. Referring to the equation $E_g = E_t + IR_a$ it is evident that if the terminal voltage E_t is to be maintained constant while IR_a is increasing, the value of E_g must be increased. Referring to Eq. (6) it will be seen that either the speed or the field flux might be increased and the generated voltage thereby raised. Increasing the field flux is the more feasible, as the prime movers used for driving generators are built to maintain a constant speed. Then as load is added to the generator, sufficient additional flux must be provided in order to raise the generated voltage enough to compensate for armature IR drop and armature reaction. In order to provide more flux, more shunt field current is necessary, so that the no-load shunt field current will be smaller than the full-load field current. There must then be some external resistance in series with the shunt field at no load, which can be cut out and the shunt field current thereby increased as load is increased.

The armature characteristic is the curve showing the relation between the external and the shunt field currents, when the terminal voltage is maintained constant by shunt field regulation. The curve is usually concave upward (as is shown in Fig. 11) due to the fact that the increase in flux per unit increase of the shunt field current, as indicated by the magnetization curve,

decreases as the iron approaches saturation. Accordingly there is needed a greater increase in shunt field current to compensate for a given value of armature IR drop and armature reaction at high loads than at light loads.

Since the number of turns upon the shunt field is constant,

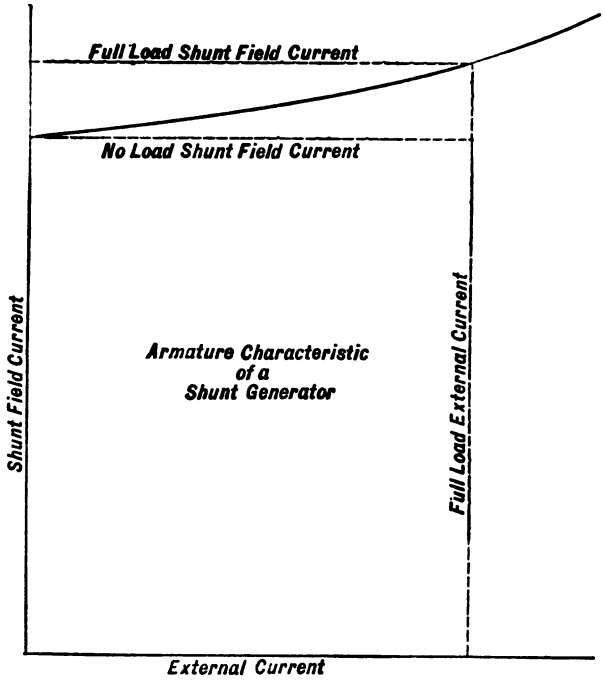

FIG. 11

and the number of shunt ampere turns therefore always proportional to the shunt field current, we can then determine the per cent increase in the number of ampere turns from no load to full load.

Connect the generator to be tested as in Fig. 10 and start at rated voltage and speed with no load upon the machine. Then add load, maintaining rated speed and adjust the shunt field resistance so that for each addition of load the voltage is brought back to

its rated value. Take eight readings up to 25 per cent overload. Use log as in Table III.

(*b*) It was pointed out in Exp. III that a shunt generator, when no attempt at regulation was made, decreased its terminal voltage with increase of load. However, if sufficient flux was added to compensate for the factors causing the decrease, then the terminal voltage remained constant. In part (*a*) it was seen that the necessary increase in field ampere-turns could be obtained by decreasing the resistance of the shunt field circuit. A very simple way of causing an increase in the field ampere-turns is to employ the external current, which causes the terminal voltage to fall, by simply passing it through the series field winding which consists of several turns of large wire. Such a machine is called a compound generator. The series field carries all or a fixed per cent of the current output of the machine and is then in series with the external circuit. The shunt field provides the correct no-load flux of the machine, while the series field compensates for the loss of voltage due to armature reaction, *IR* drop in the armature, and *IR* drop in the series field itself. If the number of series turns is more than sufficient to compensate for all the losses of E.M.F. then the terminal E.M.F. will rise with increase of load current. In lighting systems and isolated plants the overcompounding, as it is termed, is about 2 to 3 per cent, while in railway work the generators are usually 10 per cent overcompounded, the E.M.F. rising from perhaps 500 volts at no load to 550 volts at full load. This regulation is entirely automatic and almost instantaneous. Generally machines are built with more than enough series turns, and then a portion of the external current is shunted off by means of a German silver shunt, so that by varying the resistance of the shunt a wide degree of compounding can be obtained.

From the data obtained in part (*a*) the number of series turns necessary to convert the machine there used to a compound generator, can be calculated if the number of turns upon the shunt field are given, as we then know the actual increase in the ampere-turns from no load to full load. By dividing the increase in ampere-

turns by the value of the full-load external current we obtain the minimum number of series turns that the machine requires.

It will be found that the external characteristic of a compound generator is somewhat convex upward, this being due to the shape of that portion of the magnetization curve upon which the machine is operated.

In studying the compound generator it is seen that there are two different field currents, one circulating through a winding of a large number of turns of fine wire, the other through a winding of a few turns of large wire or strip, so that there must then be devised some way of expressing one in terms of the other. This is possible if we know the ratio of series to shunt turns, and this ratio can be experimentally determined as follows: The series field of the machine is first separately excited from some outside source of power and a current sent through it, equal to the full load external current of the machine. Then with the machine rotating at rated speed the voltage across the armature is determined. There being no current through the armature, generated and terminal voltages are the same. Then in turn, the shunt field is separately excited and a current is sent through it of such a value that, with the machine again rotating at rated speed, the terminal voltage is the same as before.

In both cases the armature cut the same number of lines of force, since it was rotating at the same speed and generating the same voltage. The number of ampere-turns upon the field must therefore have been identical in both cases, so that we may write

$$\phi = \frac{.4\pi n_1 I_1}{\text{reluctance}} = \frac{.4\pi n_2 I_2}{\text{reluctance}},$$

where n_1 = number of shunt turns;
n_2 = number of series turns;
I_1 = shunt field current;
I_2 = series field current.

Eliminating constants we have

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} = K,$$

which states that

$$\text{Equivalent shunt field current} = \text{series field current} \times \frac{\text{series turns}}{\text{shunt turns}},$$

or

$$\text{Equivalent shunt field current} = \text{series field current} \times K.$$

In case the series field has a German silver shunt and the actual ratio of turns is desired, it will be necessary to determine the actual current flowing through the series field. Where the value of K is to be used in getting the total magnetizing current, it is better in determining K, to assume that all of the external

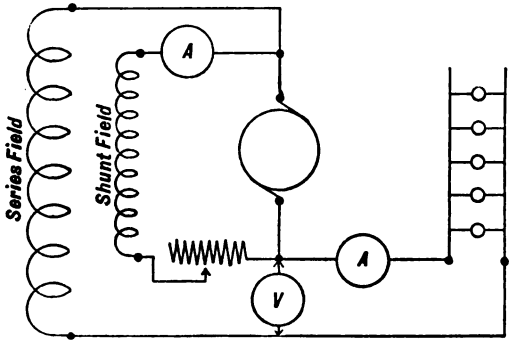

FIG. 12

current flows through the series field. The value thus obtained will be less than the actual value, but it makes calculations easier, since to obtain total magnetizing current we need only multiply the external current by K to obtain the shunt current equivalent to the series field current.

Thus for any load condition we may add to the shunt field current, the corresponding series field current multiplied by K and obtain the total magnetizing current expressed in terms of the shunt field current. If the same generator was used for both parts (*a*) and (*b*) then a curve plotted between external current and total magnetizing current from the results of part (*b*) should be very nearly the same as the armature characteristic of part (*a*).

Connect up the machine as in Fig. 12 and operating it at rated

speed, adjust the shunt field resistance to give rated no load voltage. Then with no further adjustment than to keep the speed constant, add load to the machine. Read external and shunt

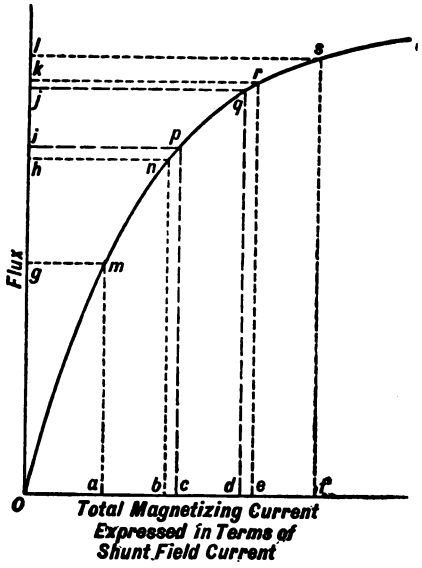

FIG. 13

field currents and terminal E.M.F. taking ten readings from no load to 25 per cent overload. Use a log as shown in table IV.

TABLE IV

Terminal Volts.	External Current.	Shunt Field Current.	Shunt Field Current Equivalent to Series Field Current.	Total Magnetizing Current.	Speed.
1	2	3	4	5	6
			(2) × "K"	3+4	

(*c*) It is of considerable importance that a compound generator be operated at rated speed if it is to compound as intended. If a flat compounded generator is operated at a speed higher than rated, but with rated voltage at no load, it will be found that it will overcompound as load is added. If it had been operated at lower than rated speed, it would have undercompounded.

We have seen from Eq. (6) that $E_g = k\phi N$, so that at rated speed there is necessary at no load a certain value of flux. This

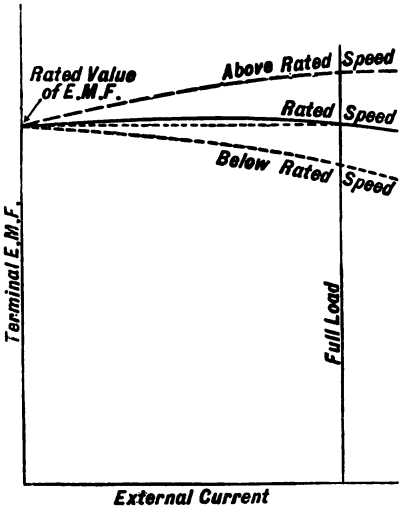

FIG. 14

is shown in Fig. 13, *oc* being the value of the shunt field current and *oi* the value of the flux produced. As load is added to the machine the series field provides a certain additional number of ampere-turns shown by the distance *cd*. The machine thus operates between the points *p* and *q* on the magnetization curve, the flux added being represented by the dstance *ij*. This amount of flux is sufficient to compensate for the loss of E.M.F. due to the addition of load, so that the machine is flat compounded, as shown in the full line curve in Fig. 14. If the machine is to be operated at a speed higher than rated, it follows that if the armature is to generate the same no-load voltage, the same amount

of flux must be cut per second, so that if the speed is raised, less flux is necessary. Under these conditions a smaller shunt field current, shown as *oa*, is sufficient, so that the machine begins operating at the point *m* on the curve, the value of the flux being *og*. When full load is put upon the machine, the series field provides the same addition of ampere-turns as before, so that the distance *ab* is equal to *cd*. The flux thereby added is shown as *gh*, which is more than *ij* (the amount just necessary to compensate for loss of voltage) which causes the machine to overcompound as shown in Fig. 14. By similar reasoning it will be seen that

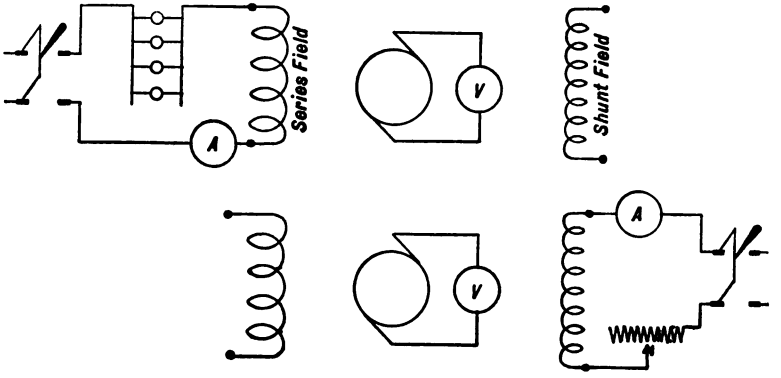

FIG. 15

operating below rated speed requires more no-load shunt field current, but the magnetic circuit is then more highly saturated so that the series field is unable to provide sufficient flux to compensatel for the losses in potential and the machine undercompounds.

Operate the machine with the same connections as in part (*b*) at speeds 15 or 20 per cent above and below rated value, beginning at the rated value of terminal voltage. Take about eight readings up to full load.

Determine the value of K for the machine as indicated in part (*b*). The connections are shown in Fig. 15.

Curves. Plot the armature characteristic and to the same set of co-ordinates, plot a curve between external current and total

magnetizing current as obtained in part (*b*). To another set of co-ordinates plot the three compound characteristics as obtained in parts (*b*) and (*c*).

Conclusions. What is the increase in ampere-turns from no load to full load in per cent of the full load value, in the shunt generator operating at rated speed? If the number of shunt field turns can be obtained, calculate the number of series field turns necessary to make the machine flat compounded with all of the external current passing through the series field. How could the number of shunt and series field turns be experimentally determined? Why should the resistance of a series field be made low? Explain why a flat compounded generator overcompounds or undercompounds, when operated at higher or lower than rated speeds respectively, if started at rated no-load voltage. Would you consider the operation of a shunt generator satisfactory for commercial work if the load is widely varying and constant terminal voltage is desired? Does the compound generator meet the difficulties? Explain why.

EXPERIMENT V

The Shunt Motor. (*a*) Speed Characteristics. (*b*) Commercial Efficiency by Brake Test.

From a structural standpoint the shunt motor and the shunt generator are identical and it will also be seen that both depend upon the same phenomena for their operation. In fact any direct-current dynamo-electric machine that can be used as a generator can also be used as a motor, the only point of difference being in their function and application. A generator has mechanical energy supplied to its shaft and this energy is converted into electrical energy by the rotation of the armature in the magnetic field. In the case of the motor, electrical energy is supplied to its field and armature and the machine converts it into mechanical form at the shaft.

Motors are usually classed according to their field windings, these being known as shunt, series, and compound; these types correspond exactly with those in the generator.

The operation of a motor involves two fundamental phenomena. The first is, that when a conductor carrying current is placed in a magnetic field, a force is developed which tends to move the conductor in a direction at right angles to itself and to the magnetic flux. This force is directly proportional to both the strength of the field and the current flowing in the conductor. The direction in which the conductor moves (for obviously it can sweep across the field in either of two ways) depends upon the direction of the current in the conductor and the direction of the magnetic flux.

In a motor we have a number of conductors distributed around the periphery of the armature, and when the latter is placed in a magnetic field, some of the conductors develop a torque when current flows through them and move at right angles to the field,

thereby rotating the armature. As they move out of the field, others take their places and in turn also move out, so that the torque is continuous and the armature keeps on rotating.

The second phenomenon is the same as that upon which the operation of a generator depends, namely, that when a conductor, whether carrying current or not, is moved in a magnetic field so as to cut lines of force, an electromotive force is generated in it.

In the case of the generator, as soon as current flows in the windings of the armature, a torque is produced which tends to move the conductors in a direction opposite to that in which the arnature rotates, due to the fact that the conductors are carrying current and are placed in a magnetic field. That is, the current, which flows through the conductors as a result of their cutting the magnetic field, causes a torque in the direction opposite to that in which they are moving. It is this counter torque which the prime mover must overcome in order to continue the rotation of the armature.

In the motor, when current is sent through the armature winding, a reaction takes place between the armature conductors and the magnetic field and the armature rotates. As soon as it begins to rotate, due to the fact that the conductors are cutting the lines of force of the field, an E.M.F. is generated, which is in the opposite direction to that impressed upon the armature. This is called the "counter" E.M.F. of the motor and evidently it cannot become equal to, or greater than the impressed E.M.F., so long as the machine is operating as a motor, for then no current would flow and the armature would cease rotating.

It follows then, that the generated E.M.F. of the generator and the counter E.M.F. of the motor are the same and that the terminal E.M.F. of the generator becomes the impressed E.M.F. of the motor. In Eq. (7) ($E_g = E_t + IR_a$), we saw that while the machine was delivering current and IR therefore positive, that E_g was greater than E_t, while if no current flowed, E_g and E_t would be equal. If the machine operates as a motor and takes current (which for the same direction of rotation could flow

in the reverse direction) IR would be negative. Under these conditions E_g is called the counter E.M.F. and is evidently less than E_t. We may then write

$$e = E_t - IR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where e = C.E.M.F.

By transposition we also have

$$E_t = e + IR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and

$$I = \frac{E_t - e}{R_a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The latter form is really Ohm's law for a motor and it indicates that the armature current is caused to flow by an effective E.M.F., which is equal to the difference between the impressed and the counter E.M.F.'s.

Since the C.E.M.F. of a motor is the same as the generated E.M.F. of a generator, then Eq. (6) is also that for the C.E.M.F. of a motor, so that we have

$$e = \frac{\phi \cdot N \cdot n \cdot 2p}{10^8 \cdot 60 \cdot c} = K\phi N, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which indicates that the C.E.M.F. depends upon the flux and the speed.

The shunt motor, as stated before, has its field and armature connected in parallel across the supply circuit. Since the resistance of the shunt field is constant except for temperature changes, the field current and therefore the flux will depend upon the line difference of potential. This usually has a constant value and as a result, the flux, so far as it depends upon the field current, will remain constant, i.e., independent of load.

With this fact in mind, let us now consider what happens when load is placed upon the armature. This means that the armature must increase its torque or turning effort and to effect this, there must be a greater force exerted between the armature inductors and the field flux. This force is proportional to the product of field flux and armature current, so that if it is to be increased there must be an increase in the armature current,

the field flux being constant. Now consider Eq. (10), in which E_t and R_a are constant. If I_a is to increase, e must decrease, and in order that this may be, since ϕ is constant, the speed must fall. This brings us to the first operating characteristic of the shunt motor, namely, that as the load is increased, the speed falls. Thus, when additional load is put upon the motor, the machine at that instant not developing the required torque, begins to slow up a trifle. This causes the C.E.M.F. to decrease, more current passes through the armature and a greater torque is developed. This continues until the motor exerts the required torque, the speed becoming constant at a value a little below what it was before. In a well-designed shunt motor the decrease in speed from no load to full load is, however, very small, so that the shunt motor is often called a constant-speed machine. From Eq. (8) it is evident that if R_a is high, the drop in speed will be large, so that to obtain good speed regulation with load, the armature resistance must be low.

Let us now consider what happens when resistance is inserted into the armature circuit. When current flows through the resistance there will be an IR drop across it, so that the E.M.F. applied to the armature terminals will be the difference between the line E.M.F. and the IR drop across the variable resistance.
Then

$$E_t - IR_{v.r.} = e + IR_a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

For this equation to be satisfied, e must decrease, but since $e = K\phi N$, in which ϕ is constant, the speed must fall. This in one sense, is equivalent to changing the armature resistance, as will be seen if we rewrite Eq. (12) in the form

$$E_t = e + I(R_a + R_{v.r.}).$$

The effect of adding resistance to the shunt field circuit is just the reverse. The first effect produced is a decrease in the shunt field current, which in turn decreases the flux and the C.E.M.F. This permits more armature current to flow and the extra torque developed accelerates the motor until the C.E.M.F.

has built up to such a value that normal armature current is again flowing. If the armature current has nearly the same value in both cases (i.e., before and after resistance was added to the field) then the value of the C.E.M.F. must be nearly the same for both cases and in order to satisfy this condition for a decrease in flux, there must be a corresponding increase in the speed. To operate the motor at a higher speed requires a slight increase in torque, so that with the weaker field the armature current must increase enough to cause the motor to exert a little greater torque than before. The practical limit to weakening the field is imposed by sparking at the brushes, due to the fact that with the weakened field, armature reaction is able to distort the field to such an extent that there is no commutating flux. The absolute limit to field weakening is complete open circuit in the shunt field. If this accidentally occurs, the machine will tend to speed up to a dangerous value, the high speed being necessary to generate the proper value of C.E.M.F., since only residual flux is present. Meanwhile it also draws an excessive value of current from the line.

When the brushes are shifted in a motor, as in the generator, armature reaction is intensified. For a backward shift, due to the creation of more back ampere-turns, part of the main field flux is neutralized and as above the speed rises. Furthermore, the effect of shifting brushes is to decrease the effective number of inductors upon the armature. In Figs. 16 and 17 is shown a motor armature in which the inductors marked with a cross carry current away from the observer, those marked with a dot toward the observer.

We have seen that when the conductors upon the armature of a motor revolve in the magnetic field, there is generated in each an E.M.F. which is in the opposite direction to that impressed upon the armature. This generated E.M.F. was called the counter E.M.F. Considering only the C.E.M.Fs., in the case of the conductors to the left of the line *aa* in Fig. 16, the ends toward the observer are positive with respect to the ends away from the observer. In those to the right of *aa'*, the observer is looking at the negative end of the conductor. In an armature

the individual conductors are always so connected that the positive end of one of the right-hand conductors is joined to the negative end of a conductor on the left of aa'. Thus all of the conductors are joined to add their individual C.E.M.Fs.

If the brushes are now shifted backwards to the position bb', as shown in Fig. 17, the current in inductors 7, 8, 15 and 16 is reversed, but as these conductors are still cutting the flux as before, the upper ends of 7 and 8 will still be negative and those of 15 and 16 positive with respect to the lower ends. Since the upper end of each conductor above bb' is joined to the upper end of

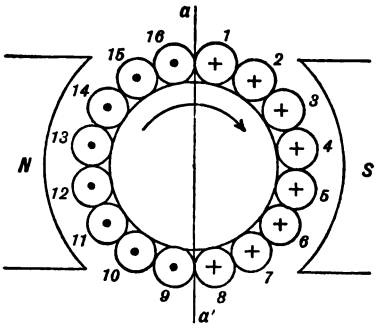

FIG. 16

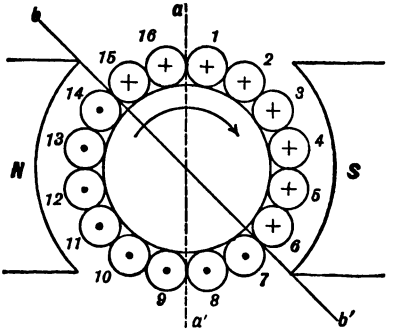

FIG. 17

some conductor below bb', then some of the conductors are connected in the improper order, i.e., the positive ends of some of the conductors are joined to a positive end of another. The C.E.M.Fs. of some of the conductors thus oppose those generated in others, and the effect is equivalent to decreasing the effective number of inductors. Since $e = K'\phi Nn$ it follows that if e is to remain at its proper value, the speed must increase.

We have seen before that the effect of additional load is to cause the speed of a shunt motor to decrease. Increase of armature current is also accompanied by greater armature reaction, whose effect, as we have just seen, is to decrease the effective flux and thereby increase the speed. It might then be argued, that if armature reaction were made great enough, a constant

speed shunt motor might be designed. Whereas this is possible, it is never done, armature reaction being kept as low as possible to obtain good commutation. Besides, the drop in speed in a shunt motor, from no load to full load, being only a few per cent., is of no great commercial importance.

In order to change the direction of rotation in a motor the relation of armature current to field flux must be changed. This involves reversing either the armature or the field current. If both are reversed, their relation remains the same and the armature continues to rotate in the same direction as before.

Connect the shunt motor to be used as in Fig. 18, choosing

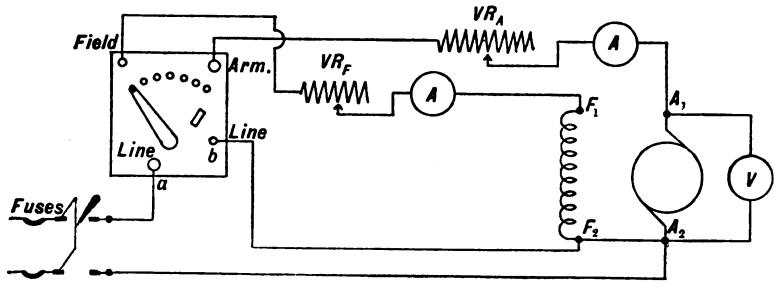

FIG. 18

ammeters and variable rheostats of suitable current capacity. The terminals of the starting box will generally be marked as in the diagram and they should be connected as indicated. The starting box shown has four terminals, of which two are marked "line"; one of these will be a large terminal (marked "*a*" in the diagram,) the other is a small terminal (marked "*b*"). Many starting boxes are provided with only three terminals, and in this case, line terminal *b* is the one omitted and the connection *b*, F_2, is left out.

With all of the resistance cut out of both rheostats, close the line switch and *slowly* move the handle of the starting box as far as it can go. If the starting box is properly connected the handle will remain in this position.

(1) With the machine running free, adjust the brushes by shift-

ing backward and forward, to obtain the minimum speed; note that at this point the sparking is a minimum. It is known as the no-load neutral point.

(2) Shift the brushes forward a small amount, note speed and sparking and read the meters. Again move them forward an equal amount and repeat. Continue this until either the speed rises 25 per cent or the sparking becomes bad. Repeat, moving the brushes backward.

NOTE: If the machine assigned is provided with commutating poles, perform 1 and 2 upon another machine.

(3) With the brushes on the neutral point, insert resistance into the shunt field circuit by means of VR_F. Do not raise the speed more than 25 per cent above the normal value. Carefully determine speeds and read all meters. Take from 8 to 10 readings, recording in a log as in table V.

TABLE V

Armature Volts.	Armature Amperes.	Field Amperes.	Speed.	Armature *IR* Drop.	C.E.M.F.

(4) Insert resistance into the armature circuit by means of VR_A (VR_F should be all cut out), again noting speed variations and reading all meters.

(5) Investigate the methods of reversing the direction of rotation of the armature.

(*b*) The rating of a motor is always given in horse-power, and is the actual or available horse-power at the motor pulley. To determine the horse-power output of a motor, we must know the pull F, which the motor is capable of exerting at the periphery of the pulley and also L, the radius of the pulley. In one revolution the point of application moves a distance $2\pi L$ with respect to the pulley, and in one minute, if N expresses the R.P.M., it moves a distance $2\pi LN$. The work done in foot-pounds per minute is

then $2\pi FLN$, and since 33,000 foot-pounds per minute is equivalent to one horse-power we have

$$\text{H.P.} = \frac{2\pi FLN}{33{,}000} = \frac{TN}{5250}, \quad \ldots\ldots \quad (13)$$

where L is expressed in feet, and T is the torque in pound-feet.

The efficiency of a motor is the ratio of the power which the motor gives out in mechanical form, to that which it receives in

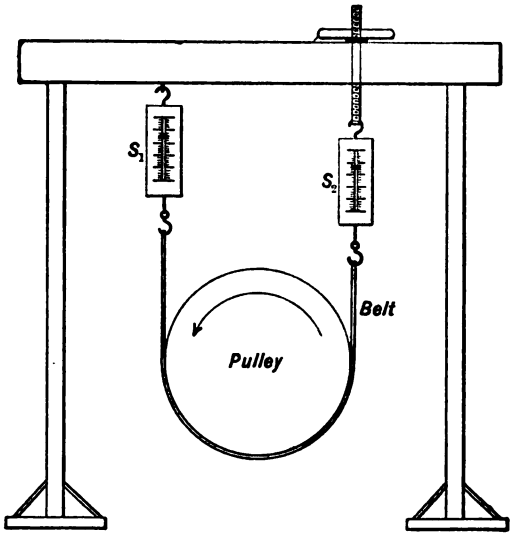

FIG. 19

electrical form. The most direct way of getting the efficiency is to measure both; to get the mechanical output a brake is most commonly used.

The principle of a mechanical brake is that mechanical energy is transformed into heat by friction. One of the simplest forms of brakes is one in which a leather or canvas belt is partly wrapped around the pulley, as shown in Fig. 19. The belt is suspended from two spring balances, one of which is suspended from a hook in some form of frame, the other being hung from a threaded rod which passes through the frame and engages a handwheel. To

adjust the brake, it is first necessary to allow the belt to hang loose, when, due to the weight of the belt there will be a small pull upon both balances. This must be noted and subtracted from all subsequent readings. Any load can now be put upon the motor by increasing the tension of the belt, which causes greater friction. In order that the pulley shall not get too hot, it is generally of the water-cooled type, i.e., built to hold water in the inside of its rim. The pull exerted by the motor is the difference between the net readings of the spring balances.

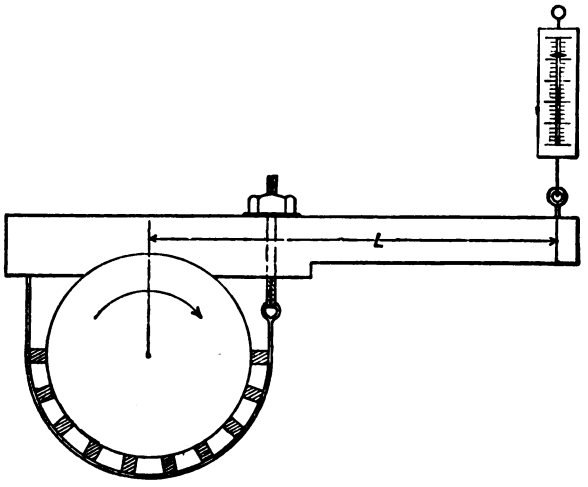

FIG. 20

Another form of brake which is very commonly used, is known as the Prony brake. A good form of it, as shown in Fig. 20, consists of a beam of wood hollowed out at one end to fit the pulley. Around the pulley are placed two thin iron straps on the inside of which are placed small blocks of wood. One end of each strap is fixed and the other ends are attached to a threaded rod which passes through the beam and can be moved up or down by means of a nut or threaded handle. The tension of the brake can thus be varied at will.

The brake described above has the effect of increasing the radius of the pulley, as the force is measured in a direction per-

pendicular to a line passing through the center of the shaft or pulley. In the form of brake described above it is rather difficult to do this, as the line from the center of the pulley to the point of application of the force is to a certain extent imaginary. Considering Fig. 21, we have that the torque is equal to the product of F and L. We have, however, that

$$L' = L\cos\theta \quad \text{or} \quad L = \frac{L'}{\cos\theta}.$$

Now $F = F'\cos\theta$. It follows then that

$$F \times L = \frac{L'}{\cos\theta} \times F'\cos\theta = L' \times F'.$$

If the arm of the brake is held horizontally, the torque of the

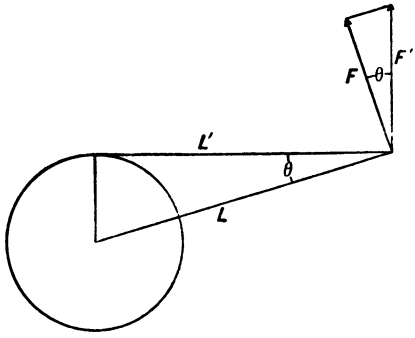

FIG. 21

motor will be given by the length of the lever arm multiplied by the pull indicated by a spring balance pulling in a direction perpendicular to the brake arm. In order to determine the pull exerted by the brake itself, clamp it fast to the pulley and slowly raise and lower the brake, taking readings upon the spring balance. The average of these two readings gives the pull due to gravity of the brake. Remember to hold the spring balance perpendicularly to the brake arm when raising or lowering it.

In Fig 22 are shown the characteristic curves of a shunt motor. It will be seen that the efficiency, at first, rises very

rapidly with increasing H.P. output, then slowly bends over, becomes horizontal and falls on overload. This is due to the fact that the losses of the machine are at the start nearly constant, but increase more and more rapidly until on overload they increase faster than the output. This is principally due to the armature copper loss, which varies as the square of the armature current. The same factors cause the curve plotted between H.P. output and amperes input, to become concave upward on overload.

To determine the commercial efficiency of the motor to be tested, connect it up as in part (*a*), omitting the two variable rheostats. Determine the full load armature current and operate the motor, first with no load upon it and then with $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{4}$ and $\frac{5}{4}$ full load armature current. Read all three meters and the two spring balances and determine the speed for each setting. Record readings in a log as shown in Table VI.

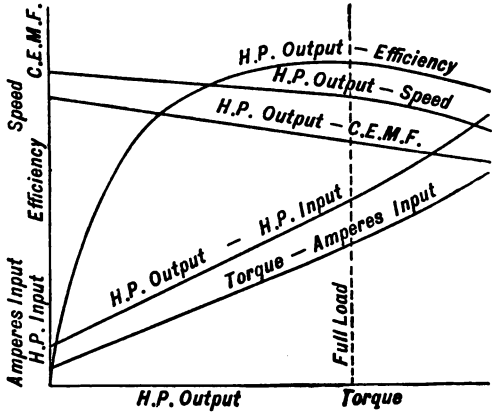

FIG. 22

Measure the armature circuit resistance for the same range of currents as used above. Use the values here determined in the calculations for both parts (*a*) and (*b*).

Curves.—(*a*) Plot a curve between field current and speed from the results of run 3. Upon the same sheet of cross-section paper plot a curve between armature E.M.F. and speed from the results of run 4. Plot speed as abscissa in both cases.

(*b*) Upon a second sheet of cross-section paper plot a set of curves as indicated in Fig. 22. Plot a curve between armature resistance and armature current upon a separate sheet.

TABLE VI

A Volts.	*B* Armature Amperes.	*C* Field Amperes.	*D* Total Amperes.	*E* C.E.M.F.	*F* Spring Balance S_1	*G* Spring Balance S_2
			$B+C$		Initial Reading =	Initial Reading =

H Net Pull.	*I* Torque.	*J* Watts Input	*K* H.P. Input.	*L* H.P. Output.	*M* Per cent Efficiency.	*N* R.P.M.
		$A \times D$				

Note—When using Prony Brake omit columns *F* and *G*.

Conclusions.—What are the principles upon which the motor operation depends? How can the speed of a shunt motor be raised? How lowered? Give the reasons for these results. What are the disadvantages of each method? Why is the curve between field current and speed concave upward?

Explain the form of the load characteristics. Why does the speed of a shunt motor fall off slightly from no load to full load? Why must the armature resistance be made low?

EXPERIMENT VI

The Motor Starting Rheostat. We have seen that when current flows through an armature that there is an IR drop, which when multiplied by the current gives I^2R, the rate of production of heat. If at ordinary temperatures the armature is unable to radiate this heat as fast as it is produced, it may rise to a temperature of 100° C. or more, at which values of temperature the various kinds of insulation upon the armature will be damaged. Accordingly the current capacity of a machine is taken as that value, which, with continuous operation will not cause the temperature of the armature to rise more than 50° C. above a room temperature of 25° C. A machine is, however, usually capable of carrying a large current for short intervals, say 150 per cent of its full load value for half an hour. It could also be possible for a motor to take an instantaneous current of many times its rated value without undue injury, but such a current value would be beyond the range of the protective apparatus, such as fuses and circuit breakers.

When the armature of a motor is at rest and an E.M.F. is applied to its terminals, there is only the resistance reaction to balance the applied voltage, so that a current will flow through the armature according to the relation

$$I_a = \frac{E_a}{R_a},$$

where I_a = armature current;
E_a = voltage impressed across the armature;
R_a = armature resistance.

The armature resistance being very low, it follows that if the value of E_a is that of the line upon which the motor is intended to operate, the armature current would be excessive.

The most natural method of preventing an abnormal current is to insert resistance in series with the armature, so that we have

$$I_a = \frac{E_t}{R_a + R_x},$$

where I_a and R_a are the same as before;

E_t = voltage of supply circuit;

R_x = external resistance in series with armature.

Let us leave this discussion for a time and consider the conditions which a motor must fulfill in order to properly start a load. In most cases a motor at starting, must exert its full load value of torque and in many cases a value greater than this. This means that the resisting torque of a load during starting of the motor, is equal to or somewhat greater than when running. The question of acceleration is also of considerable importance, for while desirable that the starting-up period be reasonably short, it must not be so short, as to demand such a value of torque which might cause a shaft to be bent or a belt to slip. It is usually found that if a motor is capable of exerting a torque of 100 to 150 per cent of its full load value, that the rate of acceleration is of the proper value.

The torque or turning effort of a motor depends only upon the current through the armature and upon the field flux, and is entirely independent of speed. It follows from this that the value of the field current should be a maximum, as then we can obtain any value of torque within the range of the motor, with minimum armature current. A shunt motor is nearly always so operated (i.e., with its field connected directly across the line) and then when full load current flows through the armature the motor exerts its full load torque. If we desire twice the full load value of torque we must supply double the armature current.

Therefore, to properly start a shunt motor we must connect its field directly across the supply line, so as to obtain maximum value of flux and then connect the armature to the supply line, in series with an external resistance of such a value that an

armature current equal to 100 to 150 per cent of its full load rating of the machine will flow.

The resistance in series with the armature also serves another purpose, namely, that of reducing the voltage applied to the armature. The effect of this (as we have seen in the previous experiment) is to reduce the speed. As soon then as the armature, with resistance in series is connected to the line, the current rises to 100 to 150 per cent of the full load value. This reacting with the field flux produces a torque which starts the armature rotating. As soon as it starts to rotate, a C.E.M.F.

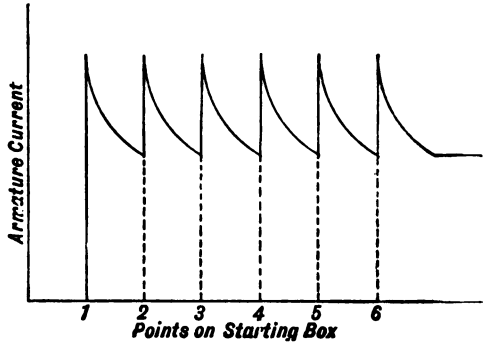

FIG. 23

is generated, which in turn reduces the current to a smaller value and the motor continues to rotate at a speed somewhat below its rated value. If now a little of the starting resistance is taken out, there will be another increase of the current, more torque will be exerted and the speed will rise to a higher value. The sudden increase in current each time the starting resistance is reduced, is occasioned by the fact that the voltage across the armature is suddenly increased, whereas the C.E.M.F. at that instant is at the value fixed by previous conditions. Finally when all of the starting resistance has been taken out, the motor is operating at full speed directly upon the line. The variations of the armature current during starting under full load are shown in Fig. 23.

Motor-starting rheostats are made up in many different forms; a rather completely equipped form is shown in Fig. 24. The

starting handle is shown in a position cutting out part of the resistance. Current enters let us say at the terminal L and flows around a solenoid B, through a switch-blade contact C, along the movable arm D to the movable arm E. The current then divides, part going through the resistances r_3, r_4, r_5 to the terminal A, which is connected to the armature. The remainder of the current goes through the resistances r_1, r_2, to the terminal F, which is connected to the field. These currents unite at M, flowing back to the line on a common conductor.

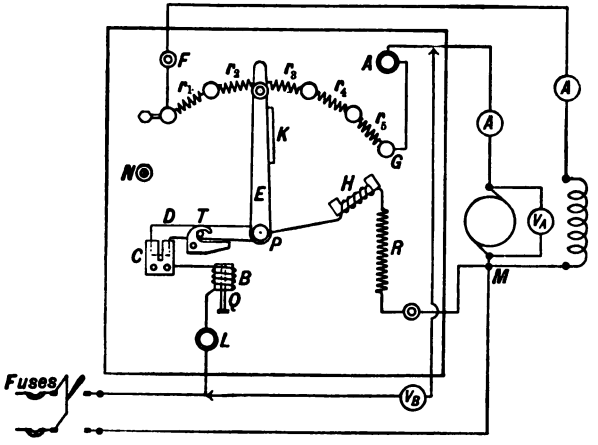

FIG. 24

When the motor is standing still, the main switch is open and the arm E of the starting rheostat is over to the left against the stop N, being held there by a spiral spring P at the pivot of the arms. To start the motor, the line switch is first closed and the arm E slowly moved over to the right against the tension of the spring P, until it is on the last notch. Here the arm is held by the attraction of an electromagnet H for the iron keeper K upon the arm. This device is commonly called the "no-voltage" release. The electromagnet is energized by current flowing from P and through the high resistance R, this circuit being thus connected directly across the supply line. When the main switch is opened or the line potential fails, the solenoid

H is demagnetized and the starting arm swings back to the starting position, automatically protecting the motor for the next starting or from a big inrush current if the line potential were again applied.

The other automatic device is the overload release, which is essentially a circuit breaker. A solenoid *B* is placed in series with the main circuit and when the motor current momentarily exceeds a certain limit, the plunger *Q* is attracted upward and strikes a trigger *T*. This releases the arm *D*, which swings in

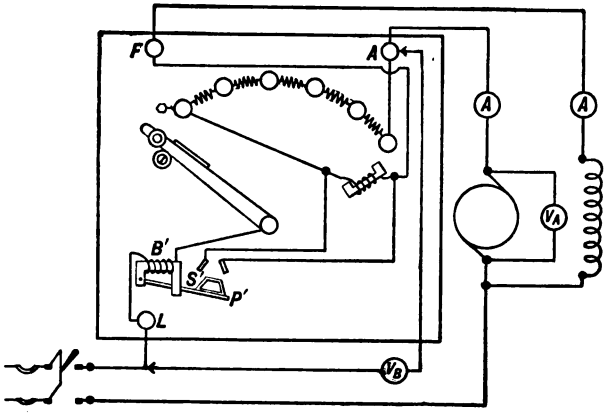

FIG. 25

toward *E*, due to the action of the spring *P*. The circuit is thus automatically opened at *C*. The lever *D* cannot be reset without moving the arm *E* back beyond its starting position.

In a majority of the starting rheostats in use, the overload device is not used, the line fuses being relied upon to open the circuit in case of overload. The advantage of having the overload release, is that it is easily reset, whereas it takes time to put in new fuses.

Another form of starting rheostat is shown in Fig. 25, in which it will be seen that the winding of the no-voltage release is in series with the field, instead of being connected directly across the line as in Fig. 24. This is an excellent arrangement, as it guards against accidental opening of the field. There is the disadvantage that a certain size starting box cannot be used

on all motors of the same size and voltage, inasmuch as the value of the field current may not be the same for the various different motors. The manufacturer of the motor can decide upon any value of field current he chooses, as field excitation is only a matter of ampere-turns. Thus in one motor the field current may not be high enough to give the electromagnet sufficient excitation to hold the starting arm, in another it may be so large as to cause the winding of the magnet to heat.

The type of overload release shown on Fig. 25 is also different from that in Fig. 24. In this case the arm P' is provided with a small piece of copper strip S', bent as shown. When P' is attracted due to an overload current flowing through the solenoid B', the copper strip makes contact with two complementary strips of copper and thus short circuits the winding of the no-voltage release. As a result the latter is short-circuited and de-energized and permits the starting arm to swing back to the starting position. The trouble with this device, though cheaper to manufacture, is that the auxiliary contacts are liable to be bent or become dirty and thus inoperative. The form of overload release shown in Fig. 24 is more positive in its action and more reliable.

For this experiment, a starting-box is necessary which will permit a motor to be operated at full load current at the various notches for short intervals without dangerous overheating. Considerable energy is transformed into heat in the box under these conditions and most types will be injured if so operated. There are, however, a few excellent types upon the market in which the resistances are imbedded in sand with no soldered connections and which are absolutely fireproof.

Connect the motor to be used in studying the operation of the starting box assigned, as in Figs. 24 or 25. The ammeter to be used in the armature circuit must have a range equal to double the full load armature current of the motor. The two voltmeters should have a range a little larger than the line voltage. Provide a slip of paper for each meter upon which the readings can be jotted down, as there will not be time to record them in the log. Three men are really needed for rapid work, one to

manipulate the starting lever and read one voltmeter and the field ammeter, the second to read the remaining meters and the third to take speed.

With no load upon the motor close the line switch and put the starting lever upon the first notch of the rheostat. At the same time the maximum throw upon the armature ammeter must be noted and immediately recorded. As soon as the motor has reached steady speed, as determined by a tachometer, a reading upon all the meters is to be taken. Then the starting lever is to be moved to the second notch, the maximum reading upon the armature ammeter, the speed and all the steady values are again to be recorded. This is continued until the running position has been reached.

Then put a brake upon the motor and with the motor running at full speed tighten the brake until the armature current is about 80 per cent full load value. Then without changing the tension of the brake stop the motor and proceed as before.

It is imperative that the readings be taken rapidly, else the box will heat excessively.

After the second run, determine the armature resistance for several values of current, covering the range recorded in the test.

TABLE VII

1	2	3	4	5	6	7	8	9
Armature E.M.F. V_A	Box E.M.F. V_B	Line E.M.F.	Armature Current.		Field Current.	C.E.M.F.	Speed.	Box Resistance.
			Maximum.	Steady.				
		1+2						

Record readings in a log as in Table VII, using steady values of armature current to calculate C.E.M.F. and box resistance.

Curves. Plot a curve between armature current and notches on starting box as indicated in Fig. 23, for both no load and full load runs. Also plot curves for both runs between speeds (abscissa) and C.E.M.F.

Conclusions. Why is a starting rheostat necessary? Why is it essential to provide maximum value of field current at the first point? Why should a motor be started slowly? How do armature and field currents vary during starting? Why cannot the usual type of starting box be used for continuous operation with the lever upon any of the starting notches?

EXPERIMENT VII

Shunt Motor Efficiency by the Stray Power Method. The electrical input to a motor is not all converted into mechanical energy, some of it going to waste in various ways. Accordingly we may write that

$$\text{Input} = \text{Output} + \text{Losses}, \quad . \quad . \quad . \quad . \quad (14)$$

and also

$$\text{Commercial Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}. \quad . \quad (15)$$

The losses in any motor can be divided as follows:

- Losses
  - Copper
    - Copper loss in armature
    - Copper loss in fields
  - Stray Power
    - Friction or Mechanical losses
      - Bearing Friction
      - Air friction or windage
      - Brush friction
    - Iron or Core losses
      - Hysteresis loss
      - Eddy current loss.

Upon consideration of the copper losses, it is evident that if we know the resistance of the armature circuit we can immediately compute the I^2R loss for any value of current. This is true for a current actually measured or assumed and the same applies to the fields.

The friction losses will vary directly with the speed within the limits between which the machine is operated. The hysteresis loss, however, varies directly as the speed and the 1.6 power of the magnetic density in the armature iron. The eddy current loss varies as the square of both the speed and the magnetic induction.

In the shunt motor as commercially operated two factors are considered constant, namely, the impressed E.M.F. and the

shunt field current. Let us, for the moment, neglect the effect of armature reaction upon the flux and therefore consider the latter as being constant under all conditions of load.

When the load upon the motor was increased, we found that there was a decrease in C.E.M.F. and a slight drop in speed. From the equation $e = K\phi N$ it follows that if the flux is considered constant, the C.E.M.F. is directly proportional to the speed.

The equation for the armature current of a shunt motor is, as we have seen,

$$I_a = \frac{E_e - e}{R},$$

so that, knowing the resistance of the armature, we are able to calculate the value of the C.E.M.F. for any value of armature current. Nor does it make any difference whether this value of armature current is one actually obtained in practice or one *assumed.* Then if we know the value of the speed and C.E.M.F. at some value of armature current, we can calculate the C.E.M.F. at any other armature current and by proportion obtain the speed. That is, we predict the speed at which the motor would run if it had a certain value of armature current. We can thus forecast the speed load curve of a shunt motor provided that we know the armature resistance and the speed at some one value of armature current.

Consider then that we know the value of armature current and speed when the motor is running free, with rated voltage applied to both armature and field. From the name-plate of the motor we can find what the manufacturer intended the full load armature current should be. From this full-load value of the armature current, we can calculate the value of the full load C.E.M.F. and by proportion obtain the full load speed of the machine, using the relation

C.E.M.F. at no load : C.E.M.F. at full load =
Speed at no load : Speed at full load

The full load speed will of course be lower than the no load speed.

Knowing the full load armature current we also know the complete power input, that being the sum of the armature and field currents multiplied by the terminal E.M.F. As we know the value of armature and field resistances, we can readily determine what the armature and field copper losses will be. So that the only other losses that so far we do not know, are the stray power losses.

When a motor is operating at no load, that is, running free, the entire input is used in overcoming losses in the machine itself. If the known copper losses be subtracted from the input, we have left the stray power loss at no load, at a particular speed and the rated value of field current. Similarly we could determine the stray power loss at any other speed and field current.

Suppose we knew the full load speed of the machine and were to operate the motor with no load upon it, at this value (which is of course lower than the no load value) and at rated value of field current. We could then calculate the stray power loss of the machine, when running at its full load speed and rated field current but with no load upon it.

The effect of armature reaction, proportional to load current and brush position, is to modify somewhat the stray power loss. If we neglect this effect, however, it is possible to obtain full load stray power or stray power at any fractional load, without actually loading the motor. If, e.g., the full load speed is known, then full load stray power may be obtained with the machine running free by measuring armature input, with normal field and sufficient impressed E.M.F. to give full load speed. Stray power loss is then obtained by subtracting the armature copper loss from the armature input. Then knowing all the losses and the total input, we can readily determine the commercial efficiency.

It is, however, necessary that we consider what effect load does have upon the value of the stray power losses. As soon as armature current flows we have armature reaction, the effect of which is generally to weaken the main field. This tends to raise the speed, of the motor, or in other words the actual speed load curve is slightly higher than the predicted curve. As a

result of these conditions the value of stray power will be slightly different from those determined with no load upon the machine. However, the difference is small and inasmuch as stray power itself is a small per cent of the total input, the error is negligible.

The advantages of the method, however, more than counterbalance this, for being a prediction method there is a great saving in power. Waste of power is unavoidable in a brake test and even at times prohibitive, as the necessary power may not be available. Then again the losses are measured by means of electrical instruments which are very accurate, very much more so than the forms of brakes used, which are also difficult to operate.

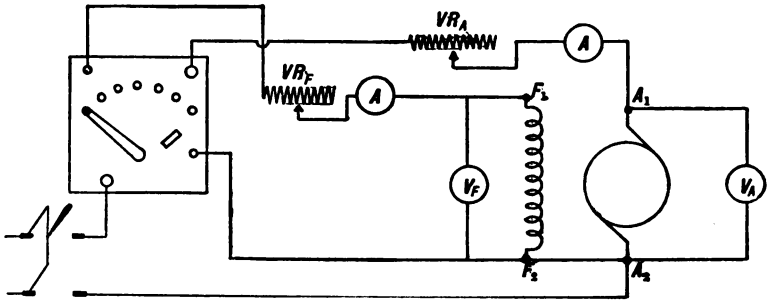

FIG. 26

The method is therefore preferable in most cases, particularly where the division of the losses into separate components is desired.

Before operating the motor to be tested first detemine the resistance of the armature circuit for 8 values of current between a very low value (2 or 3 per cent of rated value) and 150 per cent full load current, and plot the values as a curve.

Connect the motor as in Fig. 26, the source of power being preferably a laboratory generator whose terminal voltage is under control. With both VR_A and VR_F both out, operate the motor with rated voltage impressed across both field and armature. Let the machine operate for about 15 minutes and then measure speed and both field and armature currents *very* carefully.

Determine the full load armature current of the machine

from its name-plate and calculate the C.E.M.F. for 25, 37.5, 50, 75, 100, 125, and 150 per cent full load currents. Then calculate the C.E.M.F. when running light and with the speed there found, determine by proportion the speeds corresponding to the armature currents assumed above.

Then operate the machine at rated field current and at the calculated speeds and read all meters. A good method of pro-

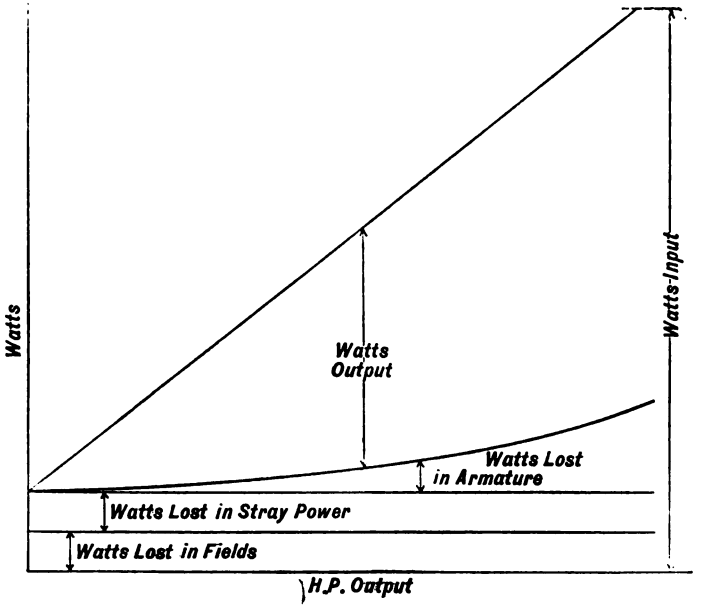

FIG. 27

cedure is to set the value of the supply line a few volts above the rated value of the machine. Then insert resistance into the field circuit by means of VR_F until rated value of field current is flowing. Then decrease the speed of the machine to the desired value by inserting resistance into the armature circuit by means of VR_A.

In the log, as shown in Table VIII, part *a* is for the operation at no load to determine the stray power loss corresponding to each load speed. All of the columns in part *b* are to be calculated or assumed except column 9, which is taken from part *a*.

TABLE VIII

a Machine Running Light

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
R.P.M. (Calculated).	Armature Current (Running Light).	Field Current (Constant).	Volts across Armature.	Volts across Field.	Watts Input to Armature.	Watts Input to Field.	Total Watts Input.	Armature I^2R Loss.	Watts Lost in Field.	Total Copper Loss.	Stray Power Loss.
					$B \times D$	$C \times E$	$F+G$	$B^2 \times Ra$	G	$I+J$	$H-K$

b. Predicted Operation under Load

1	2	3	4	5	6	7	8	9	10	11	12	13
R.P.M. (Calculated).	Rated Line Volts. (Constant).	Armature Current. (Assumed Values.)	Field Current. (Constant).	Total Current.	Watts Input	Watts Lost in Armature.	Watts Lost in Field	Stray Power Loss.	Total Losses Output.	Watts Output.	H.P. Output.	Per cent Efficiency
				$3+4$	2×5	$3^2 \times Ra$	2×4	L	$7+8+9$	$6-10$	$\frac{11}{746}$	$\frac{11}{6} \times 100$

Curves. Plot the predicted loss curves of the machine, as shown in Fig. 27; also plot the curve of predicted efficiency, using same scale for abscissa as for the loss curves. If this experiment is performed on the same machine as was previously tested by brake (Ex. 5), plot the efficiency curve obtained by brake on same curve sheet as those obtained by prediction method.

Conclusions. What is the assumption upon which the stray power method of determining the commercial efficiency of a shunt motor is based? What are the advantages of the method? How do the losses vary from no load to full load? Why does the efficiency at first rise and then fall?

EXPERIMENT VIII.

Series Motor. In the series motor, the field winding is placed in series with the armature and carries the whole armature current, and this fact causes its characteristics to be entirely different from those of the shunt motor.

The equation for the current in a series motor is as follows:

$$I_a = \frac{E_c - e}{R_a + R_{se}}, \quad \ldots\ldots\ldots \quad (16)$$

in which all of the terms are the same as before and R_{se} is the resistance of the series field.

As in the case of the shunt motor, for the series motor to increase its armature current, there must be a decrease in the C.E.M.F. In the shunt motor, where, except for armature reaction the field flux was constant, this was brought about by a slight decrease in speed. In the series motor we have a variable flux, for evidently, if the armature current increases, there will be an increase in flux.

It has been shown that for any motor $e = K\phi N$, and that the armature current can only increase as a result of a decrease in e. The increase in armature current at the same time brings about an increase in ϕ, so that if the C.E.M.F. is to fall, there must be a large decrease in speed. In the shunt motor the flux is constant and as load increases the speed falls only enough to cause a decrease in e. In the series motor, in addition to this, the speed must fall enough to counteract the large increase in flux. It would appear then from the equation $e = K\phi N$ that inasmuch as e only changes a small amount, that the speed load curve of a series motor is nearly an equilateral hyperbola. This is not quite the case, due to the fact that the iron of the field becomes saturated. When the point is reached where an increase in

armature current causes only a small increase in flux, then the speed does not fall as much as before. The speed load curve of a series motor is shown in Fig. 28. It is evident from the curve that the speed changes very materially in going from no load to full load. A very dangerous condition is, however, reached when there is no load upon the motor, for under these conditions the machine takes very little current from the line

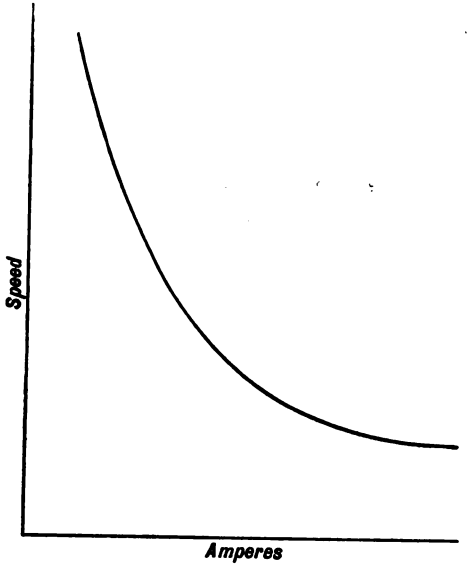

FIG. 28

and hence must have a high value of C.E.M.F. This is accompanied by a weak field and hence a dangerously high speed. For this reason a series motor is always direct connected to its load either by a coupling or through gears or chains, unless a speed limit device is attached. In the latter case, if the motor speed rises above a certain point it is automatically disconnected from the line.

The torque of any motor may be expressed by the formula $T = K\phi I_a$ and in applying this to the shunt motor we have seen that inasmuch as ϕ was nearly constant, that T varied almost

directly with I_a. In the series motor ϕ is, however, not constant, but varies with I_a. Starting with light load upon the motor we have with an increase of I_a an almost equal increase of ϕ. The machine under these conditions is operating upon the straight portion of the magnetization curve, so that the torque varies

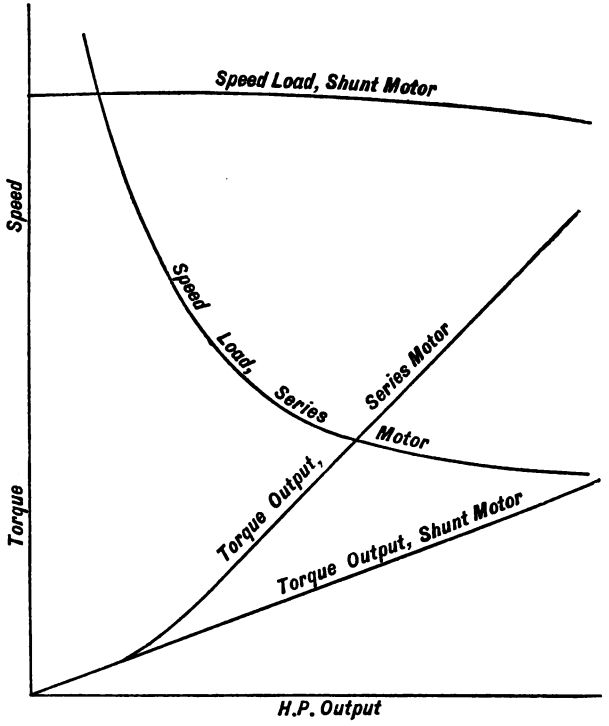

FIG. 29

as I_a^2; at the same time the motor is operating at a high speed. As the armature current increases and the saturated portion of the magnetization curve is reached, the increase in flux is no longer proportional to the current increase, so that finally the torque increases directly as the armature current.

The fact that the series motor exerts its maximum torque at stand-still, constitutes one of its greatest advantages. In Fig. 29 are shown the speed output and torque output curves

for a shunt and a series motor of the same full load rating. The speeds of both motors are the same at light loads, but at full load the speed of the series motor is very much less than that of the shunt machine.

Now since horse-power output is proportional to the product of torque and speed, i.e., H.P. output$=KTN$, it follows that in the series motor, due to its lower speed, the torque will be much greater at full load than that of the shunt motor. To exert the same torque a much larger shunt motor would be necessary.

The series motor is thus particularly valuable for work where

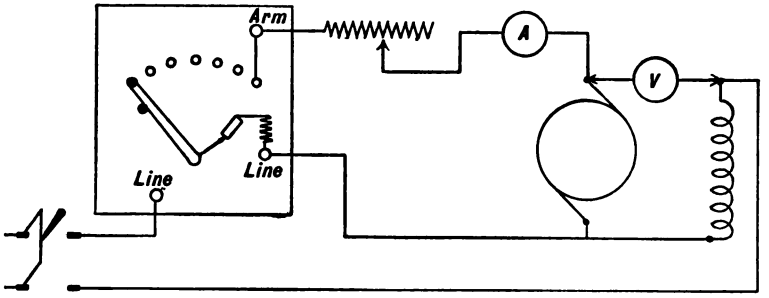

FIG. 30

a load is to be frequently started and a great starting torque is desired, as in traction service, hoists, cranes, etc. When the required torque is large the motor operates at a low speed, but where a light torque is necessary it operates at a high speed.

If the impressed voltage of the motor is decreased, the motor speed will fall and it will be found that the speed varies almost directly with the impressed voltage.

The resistances of the armature and series field are always made low for reasons of efficiency. The question of regulation is of no importance as the series motor has of itself a widely varying speed characteristic.

(1) Determine the resistance of the armature and series field, taking readings up to 150 per cent full load armature current.

(2) Connect the motor as in Fig. 30, choosing an ammeter and

a variable resistance with a capacity of about double the rated current of the motor. Start the motor and reduce the voltage across the armature to one-half of its rated value. Then starting with a very small load (no load if possible) take a reading of current and speed; then gradually increase the load by means of a brake, keeping the voltage constant at half rated value. Take eight readings up to 50 per cent overload current. If a constant potential line of one-half the rated motor voltage is available, it serves the purpose better than the use of a variable resistance. Record readings as in Table IX.

TABLE IX

E.M.F. across Motor.	Current.	Speed.	C.E.M.F.

(3) Impressing full voltage upon the motor, adjust the load until the motor takes about $\frac{2}{3}$ of its rated current. Read speed and voltage. Then decrease the voltage across the armature by about 10 per cent and adjust the current to the same value as before by means of the brake and take a set of readings. Continue to decrease the armature voltage, keeping the armature current constant until 6 or 8 readings have been taken.

(4) With rated voltage impressed upon the motor make a complete brake test up to 50 per cent overload current. If the motor is equipped with a speed limit device, allow the motor to run free and determine the speed, current, and voltage, just at the instant the speed limit device disconnects the motor from the line. If no speed limit device is provided, adjust the brake until the motor operates at 250 per cent of its *rated full load speed. Do not permit it to rotate faster than this value.*

Record readings in a form similar to Table VI.

Curves.—Plot a curve between current and the sum of the series field and armature resistances. Upon a second sheet, plot a curve between speed as ordinate and armature current from the

results of run 2, and a curve between speed and impressed voltage from the results of run 3. Upon the same sheet, from the readings of run 4, plot a speed-current curve, a torque-current and an efficiency-current curve, the current being plotted as absicssa in each case.

Conclusions. Explain why the speed of a series motor varies more widely than that of a shunt motor and why a series motor of the same horse-power rating as a shunt motor, exerts a greater full load torque. Why is the series motor better adapted to frequent starting of heavy loads than the shunt motor? Why is the torque curve concave upward at the lower end and straight at its upper end? What is the effect of lowering the terminal voltage of a series motor? Why should a series motor never be belt connected to its load?

EXPERIMENT IX

Current-Torque Curves of Different Types of Motors. This experiment is intended to show the value of the torque exerted by different types of motors with various values of armature and field currents, for purposes of comparison.

We have seen that when current flows through the armature windings of a motor whose fields are excited, there is a torque exerted, which tends to rotate the armature. The value of the torque depends upon the strength of the field and upon the value of the armature current, i.e., $T = K\phi I$.

In the case of the shunt motor we have a constant field current and if the flux remains constant we would expect that the torque would vary directly as the armature current and that a curve expressing their relation, would be a straight line. In this case the torque per ampere (i.e., the value of torque divided by the current) would be constant. This is, however, not quite the case, inasmuch as armature reaction distorts and reduces the flux so that the torque per ampere decreases with the higher values of armature current. For the same armature current and a varying field current, we do not get a straight line relation unless we are operating upon the straight portion of the magnetization curve. As the saturated portion of the curve is reached the torque does not increase as rapidly for a given increase of field current.

If to the shunt motor, a series winding is added and so connected that its M.M.F. is added to that of the shunt field, we get a cumulative compound motor. The shunt field current in this case again remains constant and the series field current increases, being the same as the armature current. The result is an increasing flux as the armature current increases, so that the torque per ampere is greater than when only a shunt winding is

used. If the machine is very much compounded, that is, if the series turns furnish a large percentage of the M.M.F. at full load, the curve between torque and current will be slightly concave upward.

In the case of the series motor, we of course have the same current flowing through both field and armature. When operating upon the straight portion of the magnetization curve, the flux is proportional to the current, so that the torque varies as the current squared:

$$T = K\phi I = KI^2.$$

Under these conditions the curve between torque and current is concave upward. As the field approaches saturation the curve gradually straightens out until at the end it becomes straight.

We are now in a position to compare the behavior of the shunt and series motor. Let us assume that we have a shunt and a series motor, each of about 5 horse-power. Let us further assume that the shunt motor has a full load speed of about 1200 R. P. M. and that the series motor has the same speed at its lightest permissible load; the full load speed of the latter would then be about 500 or 600 R. P. M. The light load speeds of the two motors would thus be the same which fact is shown in Fig. 29. Since output is proportional to the product of speed and torque, the torque exerted by both motors is about the same at light load. At full load, however, the series motor must exert double the torque of the shunt motor, as its speed is about half that of the latter. It follows that the pole pieces and armature of the series motor must be larger than those of the shunt motor to provide and accommodate the extra flux, so that the series motor is generally larger than the equivalent shunt motor.

This again brings out the great advantage of the series motor, namely, that it can exert a much greater torque at higher loads than the shunt motor for the same value of armature current.

The compound motor, as we have seen, increases its torque somewhat with load, with a greater decrease in speed than the shunt motor, but not as much as in the case of the series motor.

It is very valuable for running certain types of machine tools, as punch presses, power shears, etc., which are always provided with a heavy flywheel, the energy necessary for the actual operation of the tool, that is, say, shearing metal, etc., coming from the energy stored in the flywheel. The motor slowing up, immediately develops a strong torque and accelerates the flywheel. It is evident that the compound motor is able to accelerate the flywheel and bring it up to speed in a shorter interval of time than the shunt motor. There is also the great advantage that a relatively small motor is necessary to operate such machines provided with a heavy flywheel, since the time the machine is doing work is only a small per cent of the time it is running. In the time when the machine is running free the motor is able to store energy in the flywheel.

In order to determine the torque exerted by a motor when standing still, some form of Prony brake is clamped fast upon its pulley, as is shown in Fig. 20. When current flows through the armature and fields, let us suppose that the torque is so exerted as to tend to revolve the armature in a clockwise direction. If the brake arm be slowly moved upward against the torque of the motor, we are then, first, lifting the brake; second, overcoming the friction of the bearings; and third, overcoming the torque of the motor. This can be written as an equation as follows:

$$W_{up} = T + w + f,$$

where W_{up} = the reading upon the spring balance when the brake moves the pulley against the torque of the armature;

T = torque exerted by armature;

w = torque due to weight of brake;

f = torque due to bearing friction.

Where the brake is allowed to descend, the motor torque and the weight of the brake overcome the friction so that

$$W_{down} = T + w - f;$$

adding the two equations we have

$$T+w=\frac{W_{\text{up}}+W_{\text{down}}}{2},$$

which eliminates friction. To determine w, the weight of the brake, open the current in both armature and field and slowly raise and lower the brake, taking readings on the spring balance. The average of these two readings gives the pull due to gravity of the brake arm itself, which we have called w.

The compound motor to be used should be connected as in Fig. 31, the shunt field being separately excited and the series

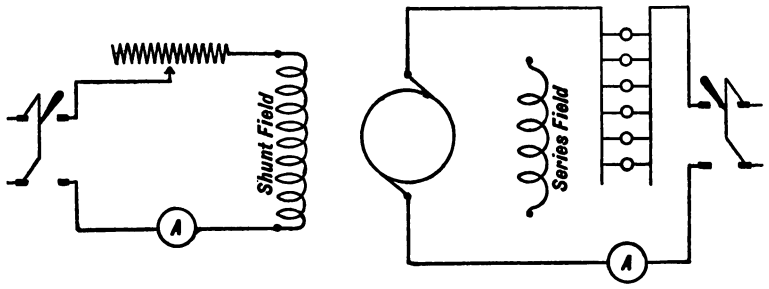

FIG. 31

winding open. The machine is thus in reality a shunt motor. By means of the variable resistance, set the value of shunt field current a trifle lower than one-half its rated value and then vary the armature current from zero up to 150 per cent full load value in about 8 steps, by varying the number of lamps burning in the lamp board, taking torque and current readings as previously noted. Keep the shunt field current constant throughout. Then raise the shunt field current to a value double that used in the first run and proceed as before. For a third run connect the series field in series with the armature as in Fig. 32, making the machine a cumulative compound motor. Vary the current through series field and armature as before with the shunt field current as in second run. Take the same readings, as before.

To determine the curves for the series motor it is preferable

to use a machine of the same voltage and H.P. output as the compound motor already tested, in order that proper comparisons can be made. Pass the same current through both armature and field as shown in Fig. 33.

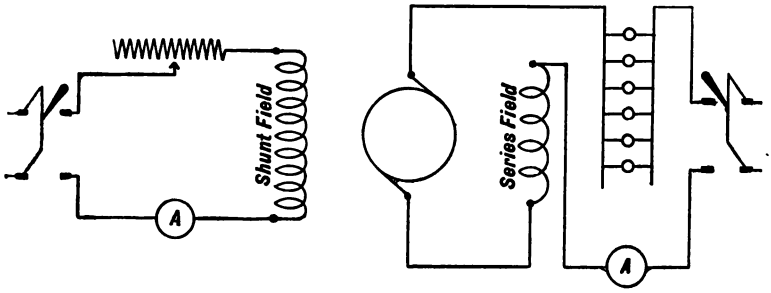

FIG. 32

Curves.—For each run plot a curve between torque and armature current (abscissa) and torque per ampere and armature current. Plot the curves for all four runs upon the same sheet of cross-section paper.

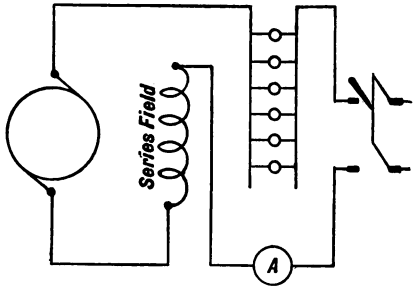

FIG. 33

Conclusions. Why are the current-torque curves for the shunt motor straight lines? Why is it concave upward at the start for the series motor? Why does torque per ampere decrease in the shunt motor and not on the compound and series motors? From the data obtained compare the action of the three types of motors.

EXPERIMENT X

Parallel Operation of Shunt Generators. In many lighting and power systems the load which the station is required to carry is much greater than the capacity of the largest direct current generator which is manufactured. It therefore becomes necessary to operate several generators on the same feeder system to supply the load. Even when the load could be carried by one large unit, it is many times preferable to install two or more smaller units to carry the load, because of the increased security against possible accident. If the load is all carried by one machine and even a minor accident happens to this machine, the station is obliged to shut down for repairs. If two or more machines are used to supply load, one of them may be shut down for repairs and the others, overloaded perhaps for a short time, will satisfactorily supply the station load. Besides, it is more efficient to operate a small machine at full load than a larger one at light load.

There are two possible schemes for connecting D.C. machines together so that they supply power to a common bus, in series or in parallel.

If they are connected in series, the voltage of the line depends upon how many machines are operating; while this type of service has been used for very special purposes, it is not feasible to use such a system for the ordinary purposes of furnishing current for light and motors. For such purposes a constant potential system is required and obviously a number of D.C. machines operating in series, their number depending upon the load, could not satisfy this requirement. Also the possible current output of such a series system is the rating of the smallest machine used.

If, however, several machines, each of the voltage required by the system, are operated in parallel in the same system, the latter will be essentially a constant potential one and the possible current output of the station depends only upon how many machines are connected to the system. The current capacity of the station is equal to the combined capacity of all the machines connected to the buses.

Although shunt wound generators are not used very much for station work, compound wound generators serving the purpose so much better, it is necessary to examine the parallel operation of shunt generators, so that the more difficult case of compound

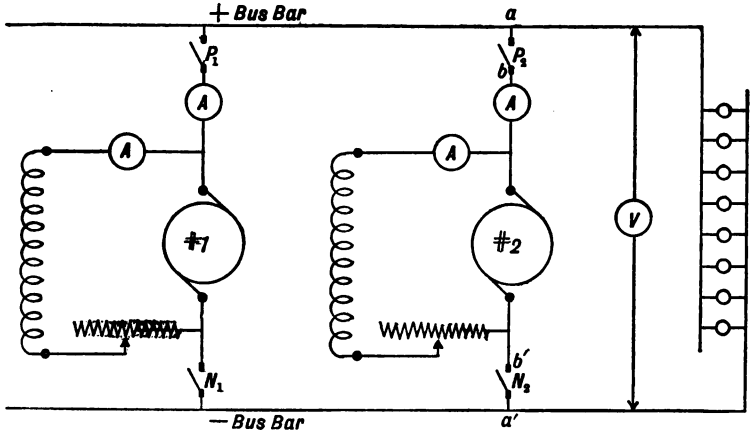

FIG. 34

generators may be understood. (The operation of compound generators in parallel will be considered in the next experiment.)

The analysis of the question may be most easily made by supposing first that one machine (say No. 1) is already connected to a load, and that whatever manipulation is carried out with the second machine (No. 2), the bus-bar voltage will be assumed constant. This latter will not generally be the case, but makes the first analysis simpler.

Consider the machines connected as in Fig. 34. Generator

No. 1 is supplying load and it is desired to connect machine No. 2 to the bus bars and divide the load between them.

Let E = voltage between bus bars;
E_g = generated voltge of No. 2;
R_a = armature resistance of No. 2;
I_2 = armature current of No. 2.

Now if the voltage E_g is exactly equal and opposite to E, it is evident that when P_2 and N_2 are closed no current will flow through the armature circuit of No. 2.

We have shown previously that

$$I_2 = \frac{E - E_g}{R_a},$$

which is also written

$$E = E_g + I_2 R_a. \quad \ldots \ldots \ldots \quad (17)$$

If when the switches P_2 and N_2 are closed, $E_g = E$, it is quite evident from the above equation, that no current will flow through the armature of machine No. 2. But if E_g is not equal to E then current will flow upon closing the switches, the *magnitude* of the current being determined by the difference in E_g and E, and the *direction* of the current depending upon whether E_g is greater or less than E. If E_g is greater than E, current will flow in the same direction as E_g tends to make it flow; i.e., the machine acts as a generator. If E_g is less than E then the line E.M.F. forces current to flow through the armature of No. 2 *against* its generated E.M.F. and No. 2 acts as a shunt motor, drawing power *from* the line instead of furnishing power to it.

If switches N_2 and P_2 are closed when E_g is not nearly equal and opposite to E, then an excessive current will necessarily flow so that the equation may be satisfied. This rush of current disturbs the line voltage, may blow fuses and circuit breakers and even injure the machine. It is always advisable, therefore, to have E_g as nearly equal to E as possible.

It has been mentioned that E_g must be opposite to E; this is sometimes expressed by saying that the machines must be con-

nected in the proper polarity. That this is a very necessary condition is almost self-evident.

Suppose that E_g were not opposite to E. As soon as switches P_2 and N_2 are closed, both E and E_g would tend to force current in the same direction, through the circuit composed of the two armatures and the bus-bars. As the resistance of this circuit is extremely low, the conditions would be one similar to a dead short circuit on each machine. To test for polarity N_2 may be closed and a voltmeter of range at least twice the voltage of the machines, connected across switch P_2. If machine No. 2 is connected with proper polarity, the voltmeter reading will be $E-E_g$; if improperly connected, the voltmeter will read $E+E_g$.

Another way of testing for polarity is to connect a voltmeter (one of range equal to machine voltage only is necessary) to the points aa' and then transfer the voltmeter leads, *without interchanging them* to the points bb'. If the voltmeter deflects the same way in both cases the polarity is correct; if not, the connections of machine No. 2 to the bus-bar switches must be reversed or else the generator must be forced to build up in the opposite direction.

The question of division of load between the two machines is now to be analyzed. When the bus-bars voltage is supposed constant and the load current is taken as constant, it is a very simple matter to see from Eq. (17) that the current, I_2, depends directly upon the value of E_g. If E_g is increased while E remains constant I_2 must increase. The output of machine No. 2 depends directly upon the value of its generated voltage.

If machine No. 2 has been brought up to rated speed, its voltage adjusted to be equal to the line voltage and the polarity test satisfied, N_2 and P_2 may be closed and no current will flow through its armature. If now its generated voltage is increased, by decreasing the resistance of the shunt field circuit, it will begin to deliver current to the line, the amount depending upon how much E_g is increased. The division of load between two shunt generators is thus always at the command of the operator; by proper field adjustment the load may be shifted from one machine to the other at will.

If the load is once equally divided, the question arises whether the division will remain equal as the load varies. This is an important point to investigate, as it is very desirable that after the machines are once adjusted for proper load division, the division should not change as the load fluctuates. If the load is once properly divided (proportionate to the relative capacities of the machines) the division will only be automatically maintained if the external characteristics of the two machines are coincident throughout the range of operation.

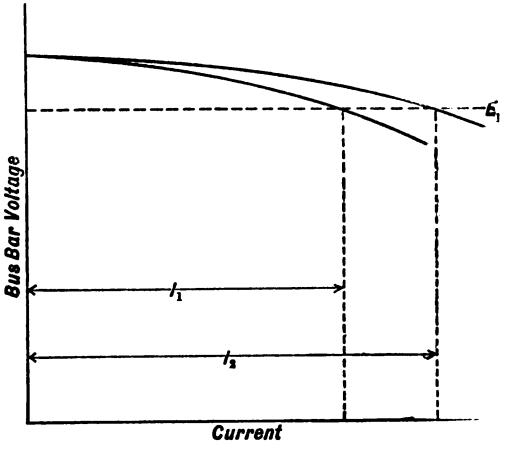

FIG. 35

Suppose the load to be zero and the voltages of the two machines adjusted to be equal. The supposed external characteristics of the machines are given in Fig. 35. Now as the machines are connected to a common bus, *their terminal voltages must always be equal.* But it is seen that if the two machines are to have the same voltage with a certain load (I_1+I_2) that machine No. 1 must be furnishing current I_1 and machine No. 2 current I_2. So that from this figure it is seen that as the load varies, (field rheostats left in fixed position) the division of load will change, the amount of change depending upon how far the external characteristics separate.

If the bus-bar voltage is allowed to vary as the load changes the division of load between the two machines will change in exactly the same manner as shown in the previous analysis, but the effects noted will not take place to the same degree. In Eq. (17) E will generally decrease as load is increased and this will tend to make machine No. 2 take more load for a given E_g than if E remained constant.

Connect the shunt generators to be operated in parallel as in Fig. 34. Attach one of the machines to the bus-bars and load it with lamps. Then bring the second machine up to speed, build up its voltage to that of the bars, and having made certain that the terminals of the machine will be properly connected as regards polarity, close the line switches. Strengthen the field of the incoming machine until it takes half of the load and then weaken the field of the first machine until it is supplying no current. Then further weaken the field of the first machine until it operates as a motor. This fact will be indicated by the armature ammeter reversing. Note that the shunt field current continues to flow in the same direction. Then bring the current in the first machine to zero and disconnect it from the bars. Continue this procedure until sufficient practice has been had, in putting machines on and off the bars and in throwing the load back and forth.

(*a*). Bring both generators in parallel upon the bars at rated voltage with no armature current in either machine. Then add load in equal steps up to the full load value of each machine, keeping it equally divided between the two machines, voltage constant at rated value and speeds constant. Read all meters, recording readings in a log as in table X.

(*b*). Starting as before with rated voltage and no load upon either machine, add load to the system but allow the total current to divide between the machines as it will. Permit the voltage to vary, but keep speeds constant at rated value.

Curves. Upon one sheet of cross-section paper plot three curves (one for run *a* and two for run *b*), between terminal E.M.F.'s (ordinate) and individual external currents. Also plot four curves (two for each run) between shunt field currents (ordinates) and individual external currents.

TABLE X

	Generator No. 1.				Generator No. 2.				Totals.	
1	2	3	4	5	6	7	8	9	10	11
System E.M.F.	External Current.	Field Current.	R.P.M.	K.W. Output.	External Current.	Field Current.	R.P.M.	K.W. Output.	Load Current.	K.W. Output.
				1×2				1×6	2+6	1×10

Conclusions. What precautions must be taken in connecting shunt generators for parallel operation? Why, if left without adjustment, do the machines not divide the load in proportion to their rated outputs? Why is the parallel operation of shunt generators rather impractical from a commercial standpoint?

EXPERIMENT XI

Parallel Operation of Compound Generators. The parallel operation of compound generators is in many respects similar to the operation of shunt generators; there are, however, a few added precautions to be noted before they may be connected to the same bus-bars.

In Fig. 36 are shown the connections of two compound generators intended for parallel operation. Let us again consider

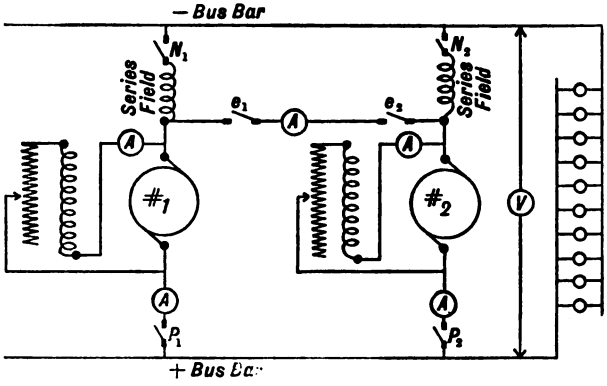

FIG. 36

No. 1 machine as connected to a load and that whatever manipulation is carried out with generator No. 2, the bus-bar voltage will again be considered constant. Let us also for the time being, suppose that the connection e_1 e_2 is not present. It is now desired to connect machine No. 2 to the same bus-bars, in order to have it share the load. The same precautions as to polarity and voltage must also apply in this case as in the operation of shunt generators and accordingly let us bring up the voltage of generator No. 2 to the same value as that of the bus-bars and close the switches

N_2 and P_2. The generator may then be made to assume load as before by strengthening its field.

The machines are, however, now operating under a condition of unstable equilibrium. Let us suppose that the speed of No. 1 machine were momentarily raised and its generated voltage thereby raised for an instant. This would immediately mean that it would take a slightly greater load, depriving No. 2 of some. The additional current flowing through the series field of the first machine would cause its generated voltage to rise still higher while that of No. 2 would be decreased. This action is li ely to continue until No. 1 is supplying all of the load and No. 2

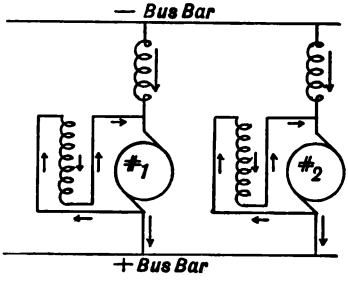

FIG. 37

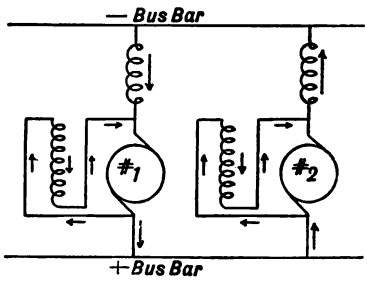

FIG. 38

none of it, and it is also likely that the generated voltage of No. 2 would drop below the voltage of the buses so that the latter would take current from the bus-bars and operate as a motor. This follows as before if we consider the equation $E=E_g-IR$.

We have seen that in the case of shunt generators in parallel, no danger would result if one machine operated as a motor. In the case of compound generators this action, with the conditions as assumed, is really dangerous. In Fig. 37 are shown two compound generators operating in parallel, both acting as generators. The current through the armature and series field flows from the negative to the positive bus as indicated by the long arrows, while the shunt field currents flow from the positive to the negative terminals of the machine as indicated by the short arrows. As the machines are compound wound generators, the M.M.F.'s of

the shunt and series fields are added and the arrows point in the same direction to indicate this fact. When No. 2 machine operates as a motor (Fig. 38), the current in its armature and series field reverses flowing from the positive to the negative bus. The current through its shunt field, however, does not reverse but flows as before, so that the M.M..F.'s of the two fields are opposed, which results in the net flux of the machine being reduced.

This in turn results in two things. First, a machine whose field is weakened tends to increase its speed; this is somewhat difficult in this case, since the machine then tends to drive its prime mover, and it is generally unable to accelerate it very much. The second result of the weakened field, is an actual decrease in generated E.M.F. or what it really is, since the machine is operating as a motor, a decreased C.E.M.F. This is due to the fact that the machine cannot raise its speed fast enough, and, besides, the action is cumulative. For if the C.E.M.F. is slightly decreased, more current flows through the series field and armature, which makes the series field stronger and the net flux weaker. Thus the more current the machine takes the more its field is weakened and so on. This goes on so rapidly that the speed has no chance to catch up and keep the C.E.M.F. up to the required value, the net result being that No. 2 forms a short circuit for No. 1, blowing fuses and circuit breakers and disturbing the system. Often the inrush current into No. 2 will be so great as to cause the series field to become so strong that it overpowers the shunt field and actually reverses the polarity of the residual magnetism of the machine.

To prevent this action and make the parallel operation of compound generators stable, a connection e_1e_2, termed the "equalizing" bus-bar, is employed, and it will be noticed that it joins the machines between their armatures and series fields. This connection prevents the reversal of the current through the series field of the machine operating as a motor, which is readily seen if we suppose switches E_1 and E_2 closed and machine No. 2 operating as a motor. Its armature current then flows from the

positive bus, but when it arrives at the point x it has a choice of either passing on through the series field of No. 2 along the negative bus and through the series field of No. 1, or flowing directly along the equalizer connection. As this latter is always of very low resistance, the current will take this path. Machine No. 2 is thus in reality operating as a shunt motor, which we have seen is not accompanied by any dangerous conditions.

As the equalizer is of very low resistance, the IR drop of the two series fields must under all conditions be equal. If the load of No. 1 increases, the IR drop of series field No. 1 tends to increase, but this can only increase if a corresponding increase in the drop of series field No. 2 takes place and this means more current through the series field of No. 2. If, therefore, No. 1 tends to increase its load and so raise its voltage, No. 2 will also raise its voltage due to the action of the increased current through its series field, which flows through the equalizer connection from No. 1. The division of the load between the two machines thus tends to remain more or less constant. It follows from the above that the resistances of the two series fields must be inversely proportional to the full load current outputs of the two machines, for if this is not so, the series field currents would not be of their proper values when the armature currents are correct. It is also necessary that the two machines have the same characteristics, that is, each must have the same degree of compounding when running separately. If they differ in this respect, the machines will not share the load equally.

The sequence of closing switches when putting machines upon the bus-bars is also very important. Again consider machine No. 1 supplying current to a load. When it becomes necessary to parallel No. 2, we might first bring it up to the bus voltage by shunt field regulation and then close E_2 and N_2. Current would then flow through the series field of No. 2, and raise its generated voltage somewhat above that of the bus-bars. This would require further shunt field regulation, so that it is better to first close switches N_2 and E_2 and then when the machine has been brought up to the bus-bar voltage by shunt field regulation, P_2 can be closed.

TABLE XI

1	Generator No. 1.					6	Generator No. 2.					Totals.	
	2	3	4	5			7	8	9	10		11	12
System E.M.F.	External Current.	Shunt Field Current.	R.P.M.	K.W. Output.		Equalizer Current.	External Current.	Shunt Field Current.	R.P.M.	K.W. Output.		Load Current.	K.W. Output.

Switch P_2 must never be closed unless the machine is up to voltage and must always be the last one thrown in.

Load is then put upon the machine by strengthening its shunt field. To remove No. 2 machine from the bus-bars, first reduce its armature current to zero by means of shunt field regulation and then open switch P_2 and then N_2 and E_2. Always open P_2 first. The prime mover of No. 2 can then be stopped and the machine shut down.

Operate the machines with the equalizer in, as was done in Ex. 10, bringing a machine upon the bus-bars, transferring the load from one machine to another and removing one machine. Also cause one machine to operate as a motor and note results.

Make two tests, taking about twelve readings in each run, including a no-load setting, and recording readings in a log as in Table XI.

(*a*) Start with the machines in parallel, but with no load upon either and gradually increase the load up to rated value, keeping system E.M.F., and speed constant at rated value and current equally divided between the two machines.

(*b*) Start as before and add load and keep only speeds constant. Allow terminal E.M.F. to vary and the load to divide as it will.

Curves. Plot three curves (one for run *a* and two for run *b*) between system E.M.F. (ordinates) and individual external currents. Upon the same sheet of cross-section plot four curves (two for each run) between shunt field currents and individual external current.

Conclusions. Explain what is liable to happen if compound generators are operated without an equalizer connection. What are the functions of the equalizer connection and how does it carry them out? What is the proper sequence of closing switches and why is this sequence absolutely necessary? Is the parallel operation of compound generators entirely satisfactory?

LIST OF A. C. EXPERIMENTS

1. Wave Shape, Power and Power Factor, Effective Values.
2. Properties of the A.C. Circuit.
3. The Alternator; its Characteristics on Non-Inductive and Inductive Load; Prediction of External Characteristic.
4. The Transformer; Operation and Characteristic Curves; Measurement of Losses and Prediction of Efficiency.
5. The Induction Motor; its Operating Characteristics with and without Added Rotor Resistances.
6. The Synchronous Motor; Synchronizing and Phase Characteristics.
7. The Rotary Converter; Effect of Speed and Voltage upon Ratio; Operating Characteristics.
8. Parallel Operation of Alternators; Distribution of Load; Circulating Current, etc.
9. Three Phase Circuits; Current and Voltage Relations; Measurement of Power.

TESTING OF ELECTRICAL MACHINERY

EXPERIMENT I

Wave Shape, Effective Values, Power and Power Factor. An alternating current is one which periodically changes its direction of flow. The frequency (number of cycles or periods per second) depends upon the class of service for which the alternating current power is to be used. In alternating current railway motors a frequency of 15 or 25 is employed; for illumination, arc and incandescent lamps, a frequency of 60 is standard; telephone currents are made up of many superimposed frequencies, ranging from perhaps 100 to 1500; for wireless telegraphy and telephony the frequency may be between 20,000 and several hundred thousand; two small metallic spheres charged with electricity of opposite kinds will, if brought close enough together, exchange and neutralize charges; the discharge current is oscillatory in character and may be of a frequency of several billions of cycles per second.

Generally an alternating current is sinusoidal in form.

If i = instantaneous value of current;
I_{max} = maximum value of current;
$p = 2\pi \times$ frequency of current;
we have

$$i = I_{max} \cos pt.$$

An alternating current generator is generally designed so that its wave form may be expressed by the formula

$$e = E_{max} \cos pt.$$

Whether or not a generator gives such a wave form depends upon the shape of the air gap between armature and pole face and also upon the distribution of armature winding.

Whenever a sine wave of E.M.F. is applied to a circuit, the current which is caused to flow is also a sine wave (neglecting distorting effects of hysteresis, dielectric loss in condensers, etc.). If the circuit is non-inductive the E.M.F. and current waves are in phase, i.e., the maximum and minimum values of the two waves occur simultaneously. If, however, the circuit offers inductance or capacity reaction, or both, the current may either lag or lead the E.M.F. in phase, depending upon which reaction predominates. These reactions are more fully analyzed in the discussion of Ex. 2.

The question arises how much power is used in a circuit in which an alternating current is flowing? The reactions which are offered to the flow of the current are of two general types, conservative or non-dissipative, and non-conservative or dissipative.

The product of the current into the dissipative reaction gives the rate at which power is being used in the circuit; the conservative reaction causes no energy loss.

If a weight suspended from a spiral spring is forced to vibrate with constant amplitude, the only force required will be that necessary to overcome the frictional resistance of the moving system. The other reactions in the system as shown to exist by the stretching of the spring (to a much greater degree than the impressed force could directly bring about) are the conservative reactions of the system; they are the forces represented by the stretching of the spring and the change in momentum of the moving weight. The power which is expended in maintaining constant amplitude of oscillation of the weight is equal to the product of the velocity of the weight into the frictional resistance of the system.

The rate at which energy is dissipated from an electrical circuit is equal, in the same way, to the product of the current into the total dissipative reaction in the system.

If i = instantaneous value of current,

R = effective resistance (see Ex. 2) of circuit, then the dissipative reaction at any instant $= iR$, and the rate at which energy is being used is equal to $i \times iR = i^2R$.

As this rate varies throughout the cycle it is necessary to get the average rate at which the power is used.

$$\text{Average power} = \frac{1}{\pi}\int_0^{\pi} i^2R\, dt.$$

Now if $i = I_{\max} \cos pt$,

$$\text{average power} = \frac{I^2_{\max}R}{\pi}\int_0^{\pi} \cos^2 pt\, dt = \frac{I^2_{\max}}{2}R.$$

Now if a direct current is passed through the same resistance the rate at which energy is dissipated is given by the formula:

$$\text{Power} = I^2R.$$

If the alternating current is varied in amplitude until it is supplying the same amount of power as is the direct current, we have

$$I^2R = \frac{I^2_{\max}}{2}R,$$

or

$$I = \frac{I_{\max}}{\sqrt{2}},$$

and this is called the *effective* value of the alternating current. It equals 0.707 of the maximum value. By the same line of reasoning the *effective* value of voltage of a sinusoidal alternating E.M.F. is equal to 0.707 the maximum value. A.C. ammeters and voltmeters are always calibrated to read *effective values.*

It is to be noticed that if the wave of current or E.M.F. is not sinusoidal, the previous integration is not so simple, but involves a number of terms of a Fourier series. The effective value of such wave is not equal to 0.707 the maximum value, but varies with the wave shape. For this reason all standard tests on alternating current apparatus must be carried out with sinusoidal wave forms; otherwise inaccuracies are likely to occur.

It has been mentioned that if conservative reactions occur in a circuit the current and voltage will not generally be in the same phase. The dissipative reaction is equal to that component of the impressed force which is in phase with the current.

If ϕ = angular difference in phase of current and impressed E.M.F.;

$E_{max} \cos \phi$ = maximum value of dissipative reaction;

$\frac{E_{max} \cos \phi}{\sqrt{2}}$ = effective value of dissipative reaction.

If I = effective value of current;

$$\text{Power supplied} = \frac{IE_{max} \cos \phi}{\sqrt{2}};$$

$$= IE \cos \phi;$$

where

$$E = \frac{E_{max}}{\sqrt{2}} = \text{effective value of impressed E.M.F.}$$

It is therefore evident that the product of E and I as obtained from A.C. meters does not generally represent the power used in a circuit. It is necessary to know $\cos \phi$ to obtain true watts. The product EI is sometimes called "apparent watts" in distinction to $EI \cos \phi$, the true power of the circuit in watts.

If we have some method of actually plotting the waves of current and E.M.F. in a circuit, in proper magnitude and phase, the power, $EI \cos \phi$, could be immediately obtained. E and I are calculated by multiplying the maximum values of the respective waves by 0.707; ϕ is measured as the angular distance between two corresponding points, say the zero points, of the two waves, remembering that a complete cycle represents 360°.

Generally it would be inconvenient and slow to use such a method, so a wattmeter is used. This instrument is connected as though an ammeter and voltmeter were in the same case; the two coils are so placed in relation to one another and the scale is so calibrated that the reading of the instrument is actually equal to $EI \cos \phi$. The current coil of the instrument is placed in series

with the circuit the power of which it is desired to measure, and the potential coil is connected in parallel with the circuit. Unlike the A.C. ammeter and voltmeter, which cannot be so connected to an A.C. circuit that the needle is deflected backward, the wattmeter may deflect backward with a wrong connection. The connection of either the current or potential coil must be reversed in such case.

It is the object of this experiment to plot curves of E.M.F. and current in an inductive circuit, in their proper phase and magnitude, and to measure with A.C. meters the E.M.F., current, and power of the circuit in order to verify the previously stated facts regarding effective values, power, cos ϕ, etc.

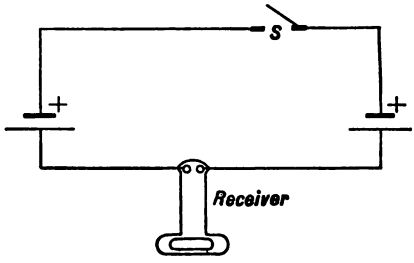

FIG. 1

The method to be employed is commonly called that of "instantaneous contact." It may be easily explained by reference to a direct current circuit. In Fig. 1 are shown two cells and a telephone receiver (any other sensitive current indicator would work as well as a telephone receiver). When switch S is closed no current will flow provided that the two E.M.F.'s in the circuit are opposite and just equal. If, however, the two E.M.F.'s are not equal and opposed, current will flow every time the switch is closed. If the switch is closed and opened at regular, short, intervals of time a more or less musical note will be heard in the telephone receiver. Suppose now this principle is applied to an alternator as shown in Fig. 2. On the same shaft with the alternator armature C, is mounted a disc of some

insulating material. A metal strip H, inserted in the disc, reaches to the periphery and is connected at its inner end to a small conducting drum and so to brush F. A brush E rests on the periphery of the disc and is insulated from the alternator frame. It will be noticed that this combination of brushes, metal strip, and insulating disc is nothing but a rotating switch, the two brushes F and E being connected together once for each revolution of the armature; moreover, whenever this contact is made, the armature occupies the same position in the field. Now the E.M.F. wave generated by the revolving armature has the same value every time the armature occupies the same posi-

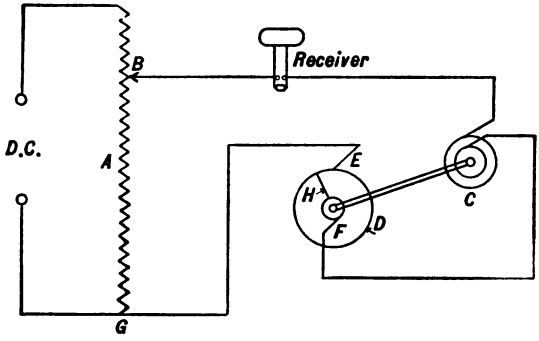

FIG. 2

tion in its field; in other words, every time the brushes F and E are connected together the instantaneous value of the E.M.F. is the same.

A variable resistance A, connected to a supply of constant voltage, serves as a source of variable potential difference; by the sliding contact B, the P.D. between G and B may be made anything desired. The local circuit, $BAGEFC$, is exactly similar in its make up to that of Fig. 1, except that the two E.M.F.'s are varying. The rotating switch closes the local circuit at the same point on successive E.M.F. waves of the alternator, shown at a, a', a'', in Fig. 3. If the slide B is moved on the potentiometer A, until the P.D. between G and B is just equal to the voltage

a, then when the rotating switch closes, no current will flow through the telephone receiver and hence no sound will be heard. A D.C. voltmeter, connected to *B* and *G*, will then give the value of the D.C. voltage necessary to balance the instantaneous value

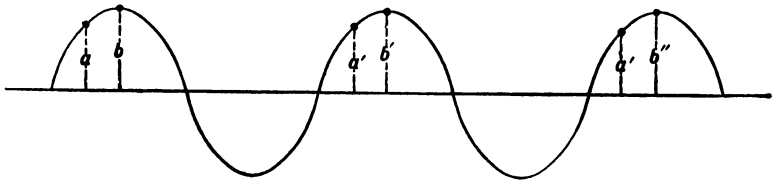

FIG. 3

of the A.C. voltage; i.e., it will give the instantaneous value of the A.C. voltage. If now the brush *E* is moved around on the periphery the local circuit will be closed when the A.C. voltage is perhaps *bb' b''* (Fig. 3). The sliding contact *B* (Fig. 2) must

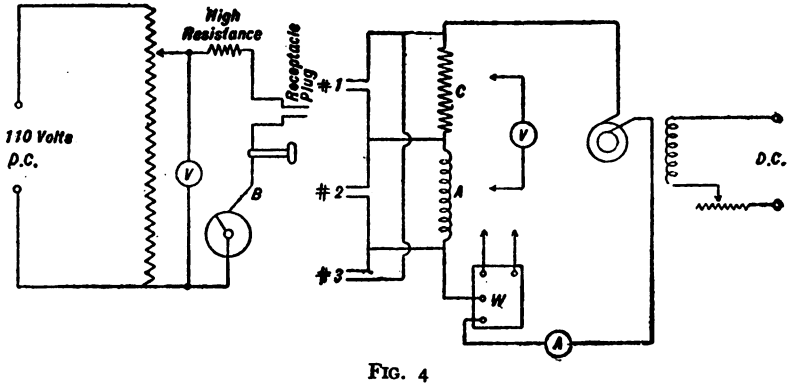

FIG. 4

then be moved to reduce the telephone sound again to zero, when the D.C. voltmeter will give the value of this voltage *b* (Fig. 3). So by moving *B* (Fig. 2) through 360 electrical degrees, taking readings at every few degrees (say 15°) the entire wave of E.M.F. may be plotted, point by point. The actual connections to be made for this test are shown in Fig. 4. The plug switch makes

it possible to connect the telephone circuit to any one of the three E.M.F. waves to be measured.

An inductive coil A and resistance C are connected in series and through the current coil of wattmeter W, through ammeter A to the A.C. generator. Taps are taken off the circuit and connected to receptacles so that the jack can connect the telephone circuit either to the terminals of A, C, or the entire circuit. For each setting of the brush B, a balance is to be obtained with the jack connected to each of the receptacles, the brush is to be moved in steps of about 15° (electrical) until the three waves are obtained for a complete cycle.

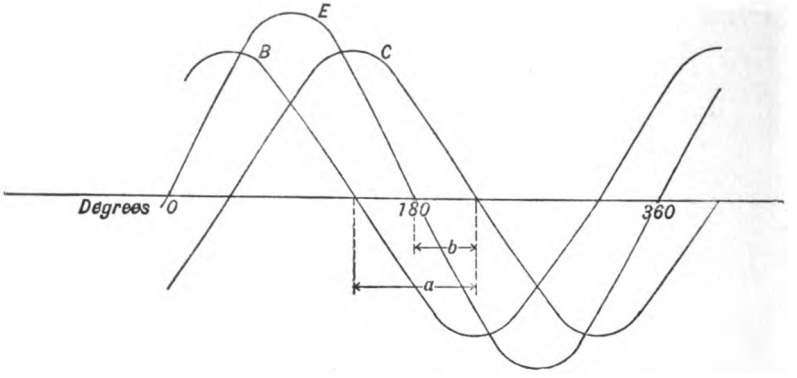

FIG. 5

The wave obtained from receptacle No. 1 gives the phase of the current, as the current and E.M.F. are in phase on a non-inductive circuit. The wave from receptacle No. 3 gives the impressed voltage, and so is the same as the terminal voltage of the alternator and shows how nearly the alternator generates a sine E.M.F.

The inductance used should have an air core, otherwise the current will be distorted and the value of ϕ obtained from the curve sheet will be difficult to interpret. After the circuit has been properly adjusted to give readable deflections on the meters used, take a reading of volts, amperes and watts for each part of

the circuit, and for the whole circuit. To get watts in any part of the circuit, connect the potential leads across that part of the circuit. Then maintain the impressed voltage and frequency constant and determine the three curves of voltage.

The impressed voltage must not be greater than about 65 per cent of the D.C. voltage used on the potentiometer resistance.

The curves obtained should look about as those given in Fig. 5. C is the drop across the resistance, hence gives the phase and form of the current; B is the drop across the inductance, and E is the impressed E.M.F. The angle of lag for the entire circuit is equal to b, measured in degrees; the phase displacement of current and E.M.F. in the inductance coil is given by a.

Check the values of cos ϕ so obtained with the values computed from readings of wattmeter, ammeter and voltmeter. With the same base and maximum value as curve E, construct a sine curve to see how closely the generated E.M.F. approximates this shape. Compare the voltmeter readings of the three parts of the circuit with the effective values as computed from the curve sheet (0.707 $\times$ maximum value).

EXPERIMENT II

Properties of the Alternating Current Circuit. The object of this test is to investigate the relations existing between the phase and magnitude of the current and impressed E.M.F. of a circuit containing inductance, capacity and resistance in series. Also the effect of frequency is to be noted and the difference between ohmic and " effective " resistance is to be measured.

When an E.M.F., either A.C. or D.C., is impressed upon any circuit, a current is caused to flow, the value of which depends upon the reactions which the circuit offers. When the characteristics of the circuit are known, the value of the current may always be found by setting up the differential equation of reactions existing in the circuit, and putting their sum equal to the impressed E.M.F. In general the reactions are proportional to the current or some function of the current, so that the differential equation involves the current, constants of the circuit, and impressed force; its solution expresses the current in terms of the impressed force and the constants of the circuit.

In the direct current circuit the solution is extremely simple, because the current itself is generally a constant quantity, but in the case of alternating currents the reactions are more complex and so the solution is more difficult. The solution will not be attempted here, but the results of the solution will be used and discussed.

The three reactions which occur in the circuit to be tested are resistance reaction, inductance reaction and capacity reaction; they will each be discussed separately.

In the ordinary D.C. circuit, as e.g., an incandescent lamp

connected to a supply line, Ohm's law expresses the relation existing between current and E.M.F.,

$$E = IR$$

where

E = potential difference at terminals of lamp;
I = current flowing through lamp;
R = resistance of lamp.

In case an A.C. E.M.F. is applied to this lamp, exactly the same law holds; E and I will be the effective values of the voltage and current while R will have exactly the same value as it had

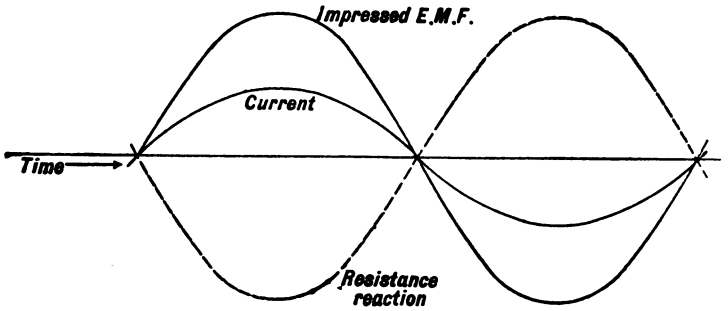

FIG. 6

for the direct current. This is the case of an A.C. circuit having only resistance reaction; moreover, the effective resistance is the same as the ohmic resistance. (This will always be true, for ordinary frequencies, unless the circuit embraces a magnetic path made up of iron. The difference will be discussed later.)

The above law gives the magnitude of the alternating current through the lamp, and its phase will be the same as that of the E.M.F. As R is a constant the current will *at every instant be directly proportional* to the E.M.F. If the voltage is a sine curve, the current will also be a sine curve (in general of different amplitude) in exactly the same phase as the voltage. The curve representation of the two variables is given in Fig. 6.

The next reaction to be considered is that offered by a coil of wire having a considerable number of turns. A coil of few turns shows the same effect as one having many turns, but not to an extent sufficient to measure with ordinary laboratory instruments. A current flowing through such a coil produces a magnetic field the strength of which is at any instant directly proportional to the value of the current. It is a property of such a magnetic field that whenever its strength is changed an electromotive force is set up in the electric circuit which embraces the magnetic field. The direction of this E.M.F. of self induction

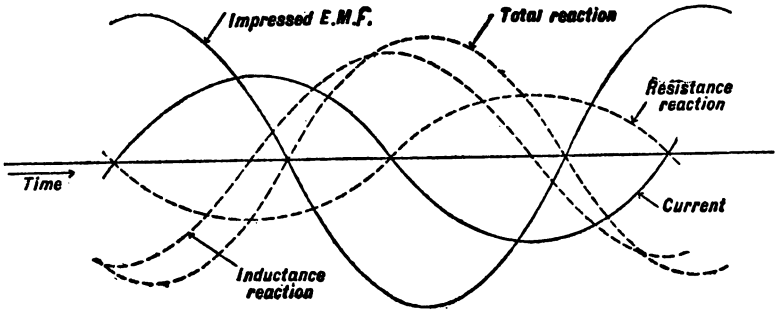

FIG. 7

depends upon whether the field is increasing or decreasing, and its magnitude depends upon the rate of change of the field and number of turns in the coil. As the field is always proportional to the current, it is evident that this inductance reaction is proportional to the rate of change of the current.

It is evident, therefore, that if an alternating current is sent through such a coil, there will be two reacting forces set up in the circuit, the resistance reaction (the coil will of course have resistance) and the inductance reaction. If the current is a sine function of the time the rate of change of the current, to which the inductance reaction is proportional, will be of similar form but displaced 90° from it. By further analysis it may be shown that this inductance reaction is 90° behind the phase of the current. The two reactions may be presented in the form

of curves as shown in Fig. 7. The current is assumed, the resistance and inductance reactions can then be plotted, and the impressed force is then found as equal to the sum of the two reactions and of course, opposite in phase. Represented vectorially it will be seen that the different forces of Fig. 7 are properly given in Fig. 8. The phase of the current is assumed as *OI*. The resistance reaction is shown as *OR* and the inductance reaction as *OX*. The combined reactions are then given by the vector *OZ*; the impressed force must be equal and opposite

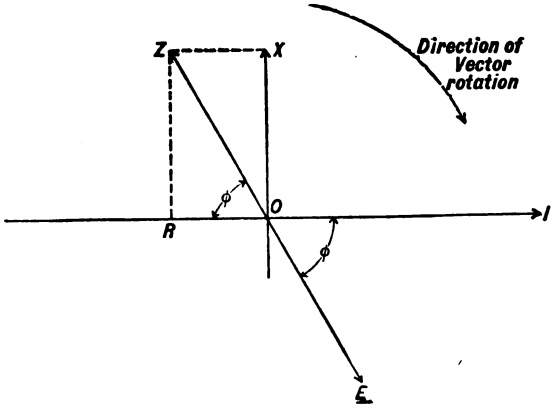

FIG. 8

to this, so is properly shown as *OE*. The length of the vector *OR* is equal (in scale of volts) to the current (in amperes) multiplied by resistance (in ohms). Maximum values of E.M.F. and current would naturally be used in constructing the vector diagram, but the diagram holds good if effective values (maximum value $/\sqrt{2}$) are used, as it simply amounts to a change in scale.

The length of the vector *OX* is proportional to *L*, the coefficient of self induction of the circuit expressed in henrys, and to the rate of change of the current. But if the current is expressed by $i = I_{max} \sin 2\pi ft$ it is at once evident that the maximum value of the inductance reaction is $2\pi fLI_{max}$. But if the other

reactions are expressed in effective values, the inductance reaction is given by $OX = 2\pi fLI$, where I is the effective value of the current.

The phase difference of the impressed force is generally designated by ϕ and from the diagram it is seen that

$$\tan \phi = \frac{2\pi fL}{R} \quad \text{or} \quad \cos \phi = \frac{R}{\sqrt{R^2 + (2\pi fL)^2}}.$$

The power used up in the circuit is equal to the current × resistance reaction. But resistance reaction = impressed E.M.F. × cos ϕ. So that power used up (expressed in watts) = $I \times IR = IE \cos \phi$.

It was proved in Ex. 1 that the wattmeter reading does give $EI \cos \phi$.

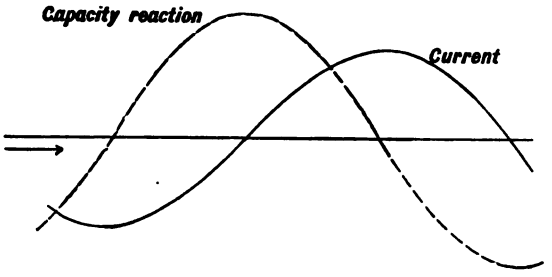

FIG. 9

If now a condenser is put in the circuit a capacity reaction will result. The counter E.M.F. of a condenser is equal to Q/C

where Q = quantity of electricity in condenser;
C = capacity of condenser.

If Q is expressed in coulombs, and C in farads, the counter E.M.F. will be given by the formula in volts.

The quantity of electricity in a condenser, of positive polarity, e.g., will be a maximum at the end of a positive alternation of the current. When the current reverses, some of the electricity begins to flow out of the condenser, so that the truth of the above statement is evident. The condenser reaction will therefore

have a maximum positive value at the end of a positive loop of current and the reaction is of the same form, with respect to time, as the current wave (if the current is a simple sine wave). The phase relations between current and condenser reaction is shown in Fig. 9, and vectorially in Fig. 10. In this latter figure OI represents the current and OC gives the condenser reaction. The magnitude of this reaction is

$$E_c = I\left(\frac{1}{2\pi fC}\right),$$

where

E_c = capacity reaction in volts;
I = current in amperes;
f = frequency;
C = capacity in farads.

The three reactions which have been discussed may now be

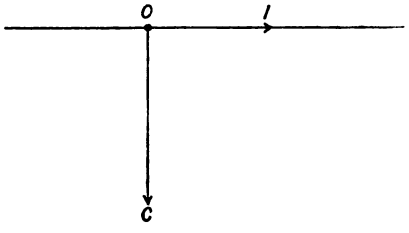

FIG. 10

grouped for convenience:

Resistance reaction $= IR$, 180° out of phase with current;
Inductance reaction $= 2\pi fLI$, 90° behind current;

Capacity reaction $= \frac{1}{2\pi fC}$, 90° ahead of current.

Generally vector diagrams do not give the reactions themselves, but the components of the impressed E.M.F. used in overcoming these reactions. These components are sometimes called the reactions; it must be remembered, however, that really the reactions are 180° out of phase with these components

of the E.M.F. In so far as no ambiguity will arise from such nomenclature and as text-books on the subject of alternating currents generally use the terms inductance drop, etc., signifying "the component of the impressed E.M.F. to balance the inductance reaction" such use of the terms will be made here. We have, therefore,

Current lags 90° behind inductance drop;
Current leads 90° ahead of capacity drop;
Current in phase with resistance drop.

The difference between ohmic and effective resistance is now to be noted. If current is sent through one coil of a transformer (a coil having a magnetic circuit in which iron is used) and the power used in the coil is measured by a wattmeter, it will be found that the wattmeter indication is much greater than I^2r where

I = the current,
r = resistance of coil, calculated from size and length of wire, or measured by direct current test.

The continually reversing magnetic field in the iron uses up energy as hysteresis and eddy current loss. This loss must be furnished by the line supplying the current, and the wattmeter measures this loss and so gives the same reading as though the ohmic resistance of the coil was much greater than it actually is.

If I = current;
r = ohmic resistance;
W = watts consumed, shown by wattmeter;
R = effective resistance;

Then $$W = I^2R;$$

Since copper loss $= I^2r$, we may write

$$W = I^2(r + x)$$

where x represents the apparent increase in resistance of the circuit produced by hysteresis and eddy currents in the iron. The value of x depends upon the quantity and quality of the iron in the magnetic circuit, the flux density in the iron and the frequency of the current.

If a condenser, inductance and non-inductive resistance are connected in series, all of the previously discussed reactions will occur in the circuit. The different reacting forces may be measured and their relative phases determined. Added vectorially these component E.M.F.'s should give the impressed E.M.F.

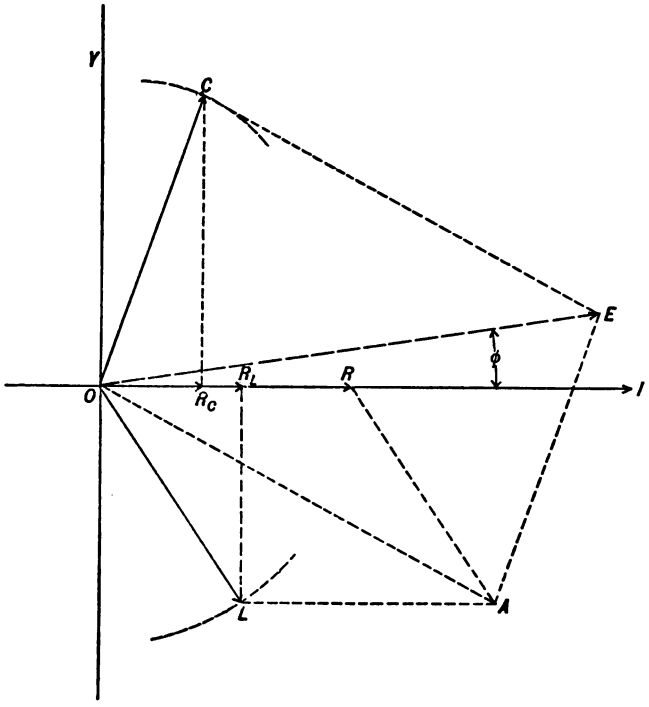

FIG. 11

both in phase and magnitude. As was shown in Ex. 1, the wattmeter reading of any part of the circuit gives the product of the resistance (effective) reaction and the current. By dividing the watts by current in the circuit the resistance reaction is therefore found. The proper phase for the voltage drop in the circuit, (or part of circuit) referred to the current, is thus obtained. In Fig. 11, is shown the vector diagram for the circuit as given

in Fig. 12. The phase of the current is assumed as OI. The non-inductive resistance drop, OR, is plotted in phase with OI and equal in magnitude to the voltage across the non-inductive resistance as indicated by the voltmeter. The resistance reaction of the inductance coil is obtained by dividing the watts used in the inductance coil by the current. This voltage is plotted in Fig. 11 as OR_L. With a radius equal to the voltage drop across the inductance coil (measured by voltmeter) an arc is drawn about O as center and the inductance drop is then plotted in such a phase as will give the requisite component OR_L in phase with the current. It is shown as OL, and the resultant of OR and

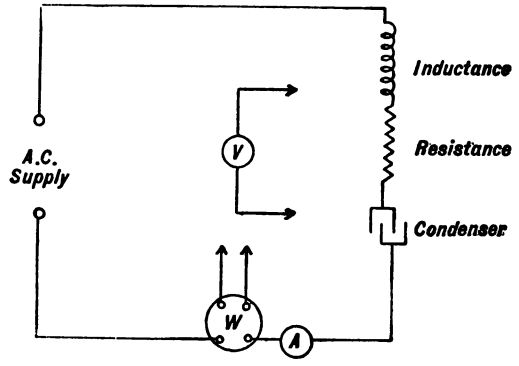

FIG. 12

OL is shown as OA. The capacity drop is plotted as OC. (It is supposed that some power is used in the condenser, giving a voltage component in phase with the current. This component is obtained in the same manner as OR_L was obtained.) The resultant of the three voltages OR, OL and OC, is shown as OE. This vector, OE, should agree both in magnitude and phase with the impressed voltage as measured and calculated from readings of voltmeter, wattmeter and ammeter.

It is to be noticed that the angle ϕ may not check very closely with the value as obtained by the formula,

$$\cos\phi = \frac{\text{watts in total circuit}}{\text{current} \times \text{impressed voltage}}.$$

This will be especially true if ϕ is small. The value determined experimentally is cos ϕ and not ϕ itself. When ϕ is small a large change in ϕ is accompanied by only a small change in cos ϕ.

With connections made as in Fig. 12, using an inductance coil with iron core, adjust the different parts of the circuit until the three drops are approximately equal. Read current, volts and watts for each part and for the whole circuit, keeping the impressed voltage constant while getting the set of readings. Change the value of the resistance (to about one-half its previous value if feasible) and take another set of readings. Keep the impressed voltage and frequency at the same values as before; the current will of course be different.

Take two more sets of readings at different frequency, using about the same values of resistance as before and keeping the same value of impressed voltage constant as before.

By direct current "drop of potential" method measure the ohmic resistance of the inductance coil and of the non-inductive resistance. Make these measurements of resistance with about the same value of current as used in the A.C. test to avoid errors due to heating.

Calculate cos ϕ, for whole circuit, for the four different runs. Construct vector diagrams for the four runs. On each diagram plot the measured value of impressed voltage for comparison with the vectorially obtained resultant.

Compare the resistances as obtained in D.C. test with those obtained in A.C. test. Explain.

What would be the proper factor of the circuit if the inductance and capacity reactances were equal?

Why does the arithmetical sum of the watts used in the different parts of the circuit equal the total watts?

Why does not the arithmetical sum of the different voltages equal the impressed E.M.F.?

EXPERIMENT III

The Alternator; its Characteristics, Measured and Predicted. The alternating current generator consists essentially of a coil of wire rotating in a magnetic field, the ends of the coil being connected to slip rings from which power is taken by means of brushes. Such a generator gives an E.M.F. which is continually reversing in direction. The air gap is so shaped and the coils so placed on the armature that the wave of generated E.M.F. is, as nearly as possible, a sine wave with respect to time.

The magnetic field of the machine is obtained from electro-magnets which must be excited from some source of direct current power. This is one feature which distinguishes the A.C. from the D.C. generator, the latter being self exciting. In an alternating current generating station several comparatively small direct current machines, called exciters, are run merely to supply the field current for the alternators.

The E.M.F. induced in an armature coil reverses every time the coil passes a field pole and so makes a complete cycle for every pair of poles passed. To figure the frequency (number of cycles per second) an A.C. generator is supplying, it is only necessary to multiply the revolutions per second by the number of *pairs of poles.*

For a given strength of magnetic field and fixed speed the generated voltage of an A.C. generator must remain constant, i.e., independent of load. As load is put on the machine with above conditions fixed the terminal voltage will, however, decrease, the decrease being nearly proportional to the load current. At loads greater than rated value the decrease in voltage is considerably greater for a given increase of load. When the load

is zero the terminal and generated voltages are equal. But when current is flowing through the armature windings, there occur in the armature coils, reactions which must be overcome by the generated E.M.F. As these reactions are proportional to the current it is evident that the terminal voltage will fall with increasing load, the generated E.M.F. remaining constant. The decrease of terminal voltage with load depends not only upon the amount of load, but also upon the kind of load, the decrease being much greater for inductive than for non-inductive loads. The reason for this will appear later. The voltage of a line to which incandescent lamps are connected should remain as nearly constant as possible. If the voltage decreases the amount of light given off decreases very rapidly, while if the lamps are operated at a higher than rated voltage, the life is materially shortened. A decrease of 5 per cent from rated voltage cuts down the amount of light from a carbon incandescent lamp nearly 30 per cent; an increase of voltage of 5 per cent above rating cuts down the life to 35 per cent of its rated value.

The feasibility of maintaining constant voltage on the line when the load is varied is an important point to investigate. That alternator giving the most constant terminal voltage with varying load, will most satisfactorily carry a lighting load and will require the least field adjustment with change of load. If rated load is put on an alternator and the field current adjusted until rated terminal voltage is obtained, then maintaining field current constant, the load is taken off, the terminal voltage will rise and the amount of this rise is a measure of the regulation of the machine. By definition,

$$\text{Regulation (in \%)} = \frac{\text{voltage at no load} - \text{voltage at full load}}{\text{voltage at full load}}.$$

The regulation of an alternator on non-inductive load will be somewhere from 5 to 15 per cent, depending upon the constants of the armature (resistance, inductance, etc.)

The first part of this experiment consists in actually loading the generator and so obtaining the external characteristic. For

non-inductive load a water barrel or lamp board may be used; for inductive load a variable inductance coil is to be connected in parallel with the non-inductive load. The connections will be as in Fig. 13, in which *A* represents the lamp board and *B* the variable inductance. For getting the external characteristic with non-inductive load switch *C* is left open, and *A*, the non inductive load is so varied that the armature current is adjusted for the desired values.

After the alternator is running at rated speed, adjust the load current and field current so that rated terminal voltage is obtained with rated current. Read the field current and maintain it at

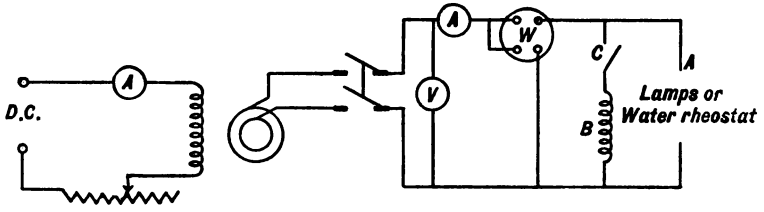

FIG. 13

this value throughout the run. Keeping speed constant, take readings of armature current and terminal voltage, with values of current equal approximately to $1\frac{1}{2}$, $1\frac{1}{4}$, rated, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ and zero load.

To get the external characteristic with inductive load (corresponding to a commercial load of transformers, induction motors, etc.), the variable inductance must be used. The method of manipulating the inductance coil is as follows. Suppose the machine is rated at 50 amperes at 110 volts and the power factor desired is 0.8.

With maximum inductance in the coil, close switch *C*, having the voltage about normal. The watts necessary for full load current at .8 power factor are $110 \times 50 \times .8 = 4400$. Adjust the lamp bank until the wattmeter reads 4.4 K.W. Then decrease the inductance of coil *B* until the ammeter A_1 reads about 50 amperes. Then adjust the field current to bring the terminal

voltage to rated value and bring A_1 exactly to 50 amperes and read field current, armature current, terminal voltage and speed (which must, of course, be at rated value). Keep field current and speed at these values and proceed in similar fashion to get points on the external characteristic at about the same values as in the previous run.

The curves should look about like those given in Fig. 14, and should be plotted in similar fashion, the inductive character-

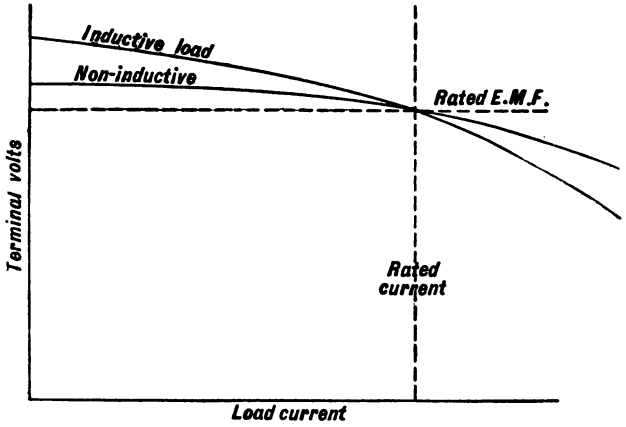

FIG. 14

istic being plotted in some manner to distinguish its points from the other curve.

To determine experimentally the external characteristic of an alternator is always more or less expensive (because of the power used) and frequently it is difficult to find proper inductances and resistances for loading. The latter consideration is very important when the alternators are of high capacity and voltage. Therefore various methods for predetermining the characteristic have been devised. One of the simplest methods will be described here.

When current is flowing through the armature it offers resistance and inductance reactions to the passage of the current.

For a given generated E.M.F. we can determine the terminal voltage by subtracting from the generated E.M.F. the two reactions in their proper magnitude and phase. The only measurements which it is necessary to make, for this scheme of predetermining the characteristic, are the resistance and inductance reactions at some known value of current (rated current preferably). These two reactions, combined vectorially with the rated terminal voltage, give the generated voltage (as explained in Ex. 2). If from this generated voltage the reactions for any other value of current are vectorially subtracted the vector remainder will be the terminal voltage for that current.

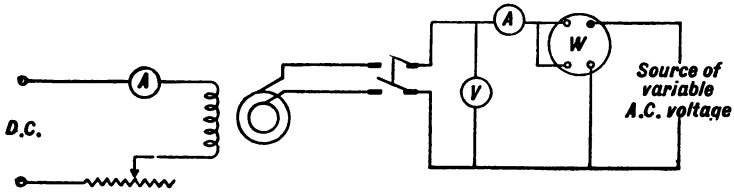

FIG. 15

With the armature stationary and connections as in Fig. 15, impress enough A.C. voltage (of frequency same as that the alternator is rated to give) to force full load current through the armature with normal exciting current in the field coils. Read amperes, volts and watts. The voltage necessary will generally be about 20 per cent of the rated E.M.F. of the machine. Take four sets of readings with the armature in different angular positions with respect to the field poles. Take the positions about 45° (electrical) apart.

Calculate the resistance reaction by dividing the average watts by current. Obtain the average impedance drop and so calculate the average inductance reaction by the formula,

$$\text{Inductance reaction} = \sqrt{\text{impedance reaction}^2 - \text{resistance reaction}^2}$$

Referring to Fig. 16, the method for predetermining the external characteristic for non-inductive load is given. The rated terminal

voltage OE_t is laid off in phase with the current OI. The resistance and inductance reactions are shown at OR and OX. The generated voltage is found as OE_g, and the locus of the E.M.F. as the load varies is the circular arc through E_g described with O as

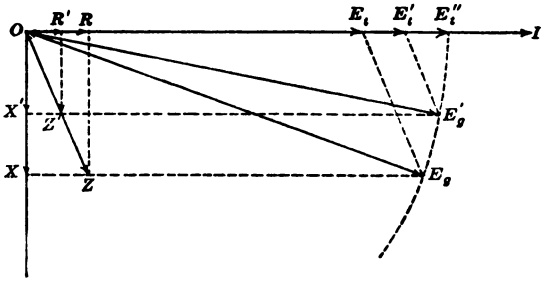

FIG. 16

center. At half load the reactions will be one-half as large (of course in same phase with respect to current as before) and the

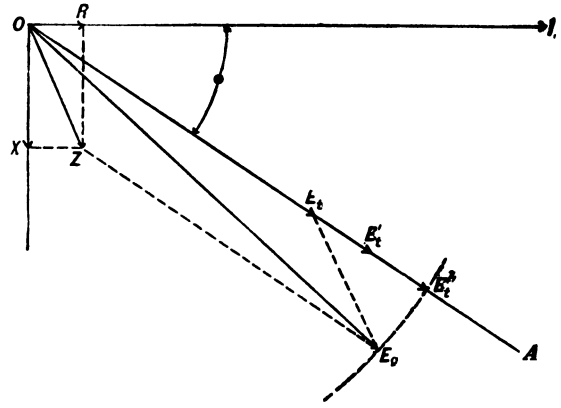

FIG. 17

generated voltage is the same. The terminal voltage is therefore found at OE_t'. At no load the terminal voltage OE_t'' is equal to the generated voltage OE_g.

For inductive load the construction is very similar. If the power factor at which the characteristic is desired is cos ϕ, the

rated terminal voltage is laid off along the line OA, making the angle ϕ with the assumed phase of the current OI. In Fig. 17 the construction is plainly indicated, so that further explanation is unnecessary.

Construct the vector diagrams for prediction of the characteristics for loads of same power factors as were used in first part of test. Indicate on the curve sheet the points as obtained from vector construction for loads of approximately those used in test.

There exists in alternating current generators another effect which has not yet been mentioned; the armature current has an influence on the field strength of the alternator, and even if the field current be maintained constant, the generated E.M.F. OE_g does not stay constant as the load is varied. For this reason the above simple scheme for predetermining the alternator characteristic is not very accurate, especially for loads of low power factors. In this short course of testing, however, it is not feasible to employ one of the more complicated methods which do take account of the armature magnetizing or demagnetizing effect.

EXPERIMENT IV

The Transformer; its Operating Characteristics; Analysis of Losses and Predetermination of Efficiency. The transformer is a stationary piece of apparatus by means of which A.C. power may be transformed from one voltage to another, either higher or lower. It finds its application where A.C. power is to be carried any considerable distance. For a given size of transmission line the power loss in the line varies as the square of the current, so that from the standpoint of efficiency the power should be at as high a voltage as the transmission line will safely carry. The voltage being high, the current will be correspondingly low and therefore the loss in the line low. At present it is not feasible to operate transmission lines at higher than 110,000 volts; if a higher pressure than this is used the losses by leakage over insulators and actual leakage currents into the surrounding atmosphere become so great, that the efficiency of transmission begins to decrease because of these losses.

A.C. generators are not economically built for E.M.F.'s exceeding perhaps 15,000 volts. At the point where power is used for motors, lights, etc., the required voltage is generally less than 440 volts. A possible problem then is to generate at perhaps 10,000 volts, transmit at 100,000 volts and utilize at 200 volts. To accomplish these changes in voltage is the function of the stationary transformer. At the beginning of the line a "step up" transformer with ratio 1 to 10 would be used; at the end of the line a "step down" transformer of ratio 500 to 1 would be used. The latter transformation might possibly be carried out in two steps: the transmission line might be connected to the distributing feeders with 50:1 transformers, and

the supply for lights, etc., be take off the distributing feeders by step down transformers of ratio 10:1. The difference between a "step up" and a "step down" transformer is merely one of service; a 10 K.V.A. 1100–110 volt transformer is one which will transform 10 kilo-volt amperes from 1100 to 110 volts if used as a "step down" transformer or will transform the same amount of power from 110 volts to about 1100 volts, when used as a "step up" transformer.

A transformer consists essentially of a closed iron magnetic circuit upon which are wound two insulated coils of wire. The two coils are generally wound in sections, the different sections being so interspersed that the magnetic leakage between the two coils is a minimum. On open circuit (i.e., the transformer supplying no load) the ratio of voltages is equal to the ratio of the numbers of turns of the two coils; as load is put on the transformer the terminal voltage of the secondary will decrease from this value. The name "secondary" is applied to that coil from which power is taken; the coil connected to the power supply line is termed the "primary."

The operation of a transformer is essentially as follows: When the primary coil is connected to a source of A.C. power, a current will flow in the coil, and if there were no hysteresis in the iron core this current would be of the same form as the E.M.F. wave of the source of supply. When hysteresis occurs to any appreciable extent in the iron core this exciting current is distorted in form, but alternates with the same frequency as the impressed E.M.F. The alternating current produces in the iron core an alternating magnetic field. Now any other coil threading the alternating magnetic field has induced in it an alternating E.M.F., the magnitude of which E.M.F. depends upon the number of turns in the second coil. The secondary E.M.F. has the same shape as the E.M.F. impressed on the primary.

The exciting current (primary current with secondary open circuited) is only a few per cent of the full load rated current of the transformer. When the secondary is loaded with a certain current a corresponding current flows into the primary because

of the reactions occurring between the two coils, the reactions being brought about by the magnetic field which is common to the two coils.

If a constant E.M.F. is impressed on the primary the secondary terminal volts decrease somewhat with increase of load; the decrease is caused by the resistance and inductance reactions which occur in the transformer itself.

Three characteristics to be investigated in this test, are efficiency, primary power factor, and secondary terminal volts, the load to be non-inductive and the primary impressed E.M.F. to be maintained constant at rated value.

The efficiency will be found to rise very rapidly with the load and will be practically constant between $\frac{1}{4}$ full load and $1\frac{1}{4}$ full load. There are no moving parts to the transformer and hence no mechanical losses to be supplied. This feature makes the transformer the most efficient of all pieces of A.C. apparatus. The full load efficiency of a transformer may be between 92 and 98 per cent, depending upon the capacity, being greater for the larger sizes.

At no load the power factor is very low, perhaps 0.3 or 0.4. As load is put on, this rapidly increases and from $\frac{1}{4}$ load up will be between .98 and 1.0, these figures being for non-inductive load on the secondary. At no load the secondary voltage is equal to the impressed E.M.F. multiplied by the ratio of the numbers of turns on the primary and secondary. As load is applied the secondary terminal E.M.F. gradually decreases, the amount of decrease depending upon the amount of load and whether or not it is inductive. For non-inductive load the decrease from no load to full load may be about 5 per cent or less.

In obtaining these three characteristics by loading, make connections as in Fig. 18. By the arrangement of two double throw and two single throw switches as shown, only one set of meters is required. The most important reason for using this switching arrangement, however, is to get rid of calibration errors in the instruments. The efficiency and power factor being so high, the different meters have to be very accurate, if absurd

results are not to be obtained (efficiency greater than 100 per cent, etc.). The voltmeter V_1 is used merely to maintain the impressed voltage of the transformer constant. The input in volts, amperes and watts is obtained by having S_1 and S_2 thrown to the left, S_4 open and S_3 closed. To get output, S_4 is first closed, S_1 and S_2 thrown to the right, then S_3 opened. The load is fixed at the desired value, then while V_1 is maintained at the rated value of the transformer, both input and output are read.

As the load is non-inductive, the wattmeter reading of output should equal volt-amperes output. If it is not so, a wattmeter

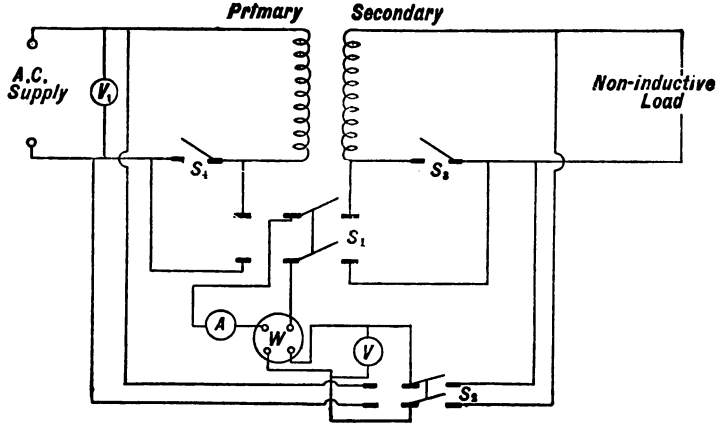

FIG. 18

calibration curve is to be constructed, using wattmeter reading (of secondary load) as one ordinate and volts × amperes as the other. Calling the voltmeter and ammeter correct in their calibration, the correct wattmeter reading is thus given in terms of their readings. From this calibration curve, correct the readings of input watts. The efficiency is then obtained as the quotient of watts output to watts input, corrected readings being used. The power factor of the primary is obtained as quotient of watts (corrected) by volt-amperes of primary. The external character-

istic is obtained directly from the readings of secondary E.M.F. and current.

There are two different losses which occur in the transformer: hysteresis and eddy currents in the iron core, due to the continually reversing magnetic field, and the ohmic or copper loss, due to the currents flowing through the windings of the transformer. The iron loss is practically independent of load because the strength of the magnetic field in the core changes but slightly from no load to full load. It is to be regarded as constant in this test. When there is no load on the transformer there is in the primary coil merely the exciting current flowing; the magnitude of which varies between 2 and 5 per cent of full load current. In the secondary coil there is no current at all; when it is remembered that the copper loss varies with the (current)^2 it is readily appreciated that the copper loss of the transformer with open secondary circuit is entirely negligible. The iron loss is, however, normal. The no load input is therefore taken as being all iron loss; the loss is measured by using a suitable wattmeter in the primary and impressing normal E.M.F. at rated frequency; *the secondary is to be open.*

The wattmeter to be used for obtaining this reading must have a potential capacity equal to rated E.M.F. of the transformer and a current coil of capacity equal to about 5 per cent of the current rating of the transformer. *This wattmeter is to be used only for the iron loss test;* be sure and remove it from the circuit before making the copper loss test which is described below. Because of the retentiveness of the iron magnetic circuit of the transformer, the current which is taken when the transformer is first switched to a line of rated voltage may be excessive and generally will cause damage to the wattmeter. To get rid of this possibility, before connecting the transformer to the line reduce the voltage of the line supplying the power to as low a value as possible, switch the transformer to the line and then gradually bring up the voltage to normal. Only one reading of the iron loss is necessary.

The copper loss may be put equal to $(I_p^2R_p+I_s^2R_s)$ where I_p and I_s represent the primary and secondary currents and R_p

and R_s represent the two resistances. $I_p = aI_s$, where a is the ratio of the transformer; also $R_s = a^2 R_p$.

Therefore copper loss $= I_p^2 R_p + I_s^2 R_s$;

$= I_s^2 R_s + I_s^2 R_s$;

$= I_s^2 (R_s + R_s)$;

$= I_s^2 R$;

where R is what we may call the equivalent resistance of the transformer. If the copper loss is measured for any value of I_s, R can be computed. If connections are made as in Fig. 19, the copper loss may be measured. With the secondary short circuited only a very small impressed E.M.F. is necessary to cause full load current to flow in the secondary circuit. If then

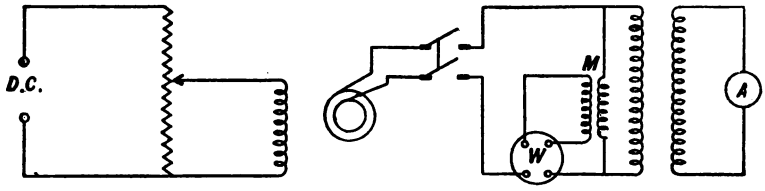

FIG. 19

the wattmeter reading is taken, it represents the copper loss in both coils for full load secondary current. It is well to excite the alternator field by a potentiometer connection to the D.C. line; the voltage may be reduced as low as desirable with this scheme of connections. The wattmeter used in this test must have a current capacity *equal to full load current of the primary*. It will likely be necessary to use a small potential transformer M to get sufficient E.M.F. on the wattmeter to give a readable deflection. The wattmeter reading must then be divided by the ratio of M.

A small iron loss is incurred in this test, but it is so small as to be negligible. The iron loss varies nearly with the impressed E.M.F. to the 1.6 power and so is very small when the impressed E.M.F. is small.

After R is computed from the copper loss test, the curve of I^2R may be plotted by taking suitable values of I. The two loss curves have the form given in Fig. 20, and they may be used to predict quite accurately the efficiency of the transformer for different loads. The total loss curve is plotted as the sum of the iron and copper losses. Suppose the curves represent the losses in a transformer whose secondary voltage is 100 (assumed constant for this calculation with practically no error involved.) At 40 amperes output, non-inductive load, the watts output=

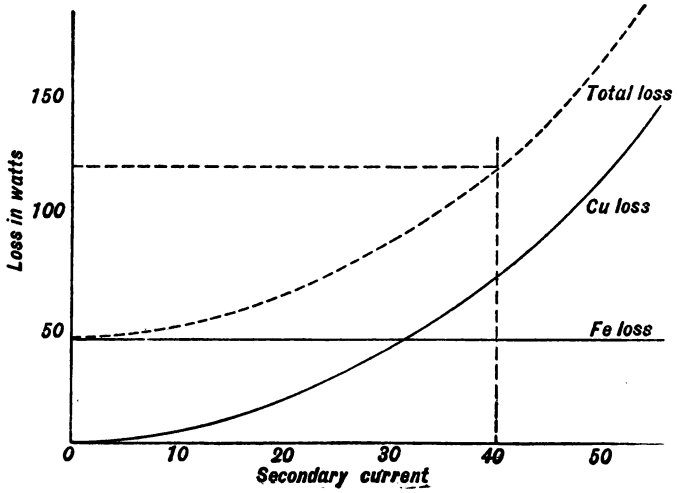

FIG. 20

$100 \times 40 = 4000$. From the loss curve it is seen that with 40 amperes output the loss is 115 watts. The input must therefore be $= 4000 + 115 = 4115$ watts. The efficiency = output/input $= \frac{4000}{4115}$. The efficiency for any other output may be similarly computed.

Make the load test called for and plot the three characteristics on one curve sheet with current output as abscissa.

Measure the two losses as described; plot the results on a second sheet and calculate the efficiency for several outputs.

Plot the efficiency points so obtained on curve sheet No. 1, to see how nearly the predicted values of efficiency agree with the measured values. If discrepancies occur it is likely that the predicted values are the correct ones, as this prediction method is more accurate than the actual measurement.

The other characteristics may also be predetermined, as was the efficiency, but it is not thought well to further complicate the test.

EXPERIMENT V

The Induction Motor; its Operating Characteristics with and without Added Rotor Resistance. For such purposes as require a driver of practically constant speed, when alternating current power is available, the induction motor is nearly always used. Its speed is not quite constant, but decreases as the load is increased; the decrease in speed, or " slip," as it is called, may

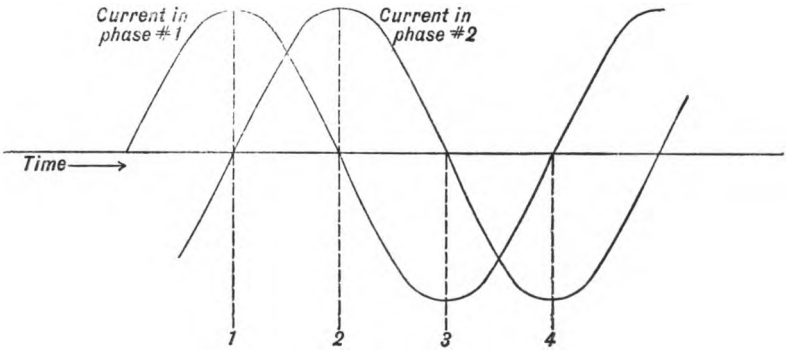

FIG. 21

be between 5 and 8 per cent at full load, the slip being expressed in percentage of synchronous speed.

Nearly all induction motors are polyphase, i.e., they are fed from a network of conductors, from which currents of different phases may be taken. Of all polyphase systems the three phase is most important, but as the two phase motor serves as well for analysis as the three phase and is somewhat simpler to represent, it will be described here.

A two phase generator is one having usually two entirely sep-

arate coils. The coils are identical in every respect except their position on the armature, one coil being placed 90° (electrical) behind the other. The two coils are connected each to two slip rings and the two phase power is distributed on four wires. (Three rings and three wires may sometimes be used.) If two similar loads are connected to the two phases, the currents in these load circuits will have the form and phase relations given in Fig. 21.

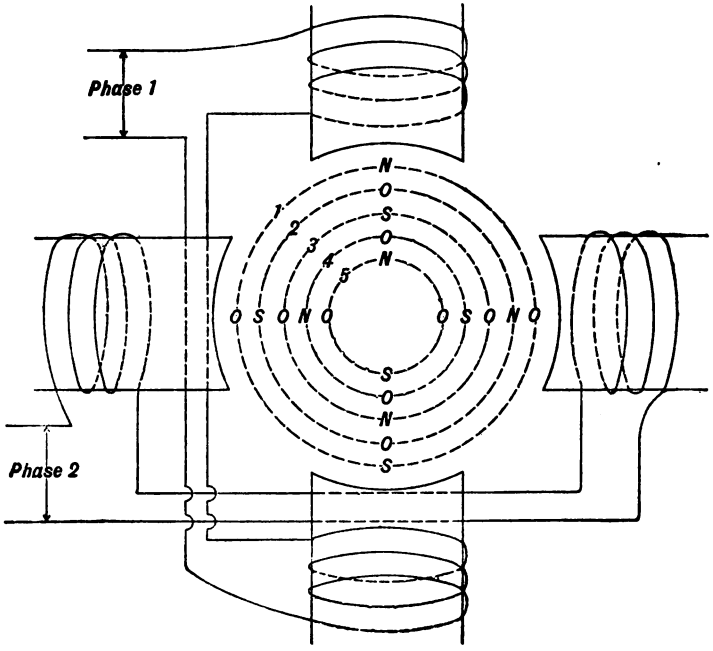

FIG. 22

Suppose now that we have a two phase induction motor with the stator (stationary part) wound with two sets of coils on poles as shown and that the two coils are connected to a two phase system as shown. By reference to Fig. 22, the polarity of the magnetic field of the motor may be determined at any time, and in Fig. 22 this polarity is represented for the different times

shown in Fig. 21, by the letters N, S, O, on the different circles. Circle numbered 1 shows the polarity of the field at time 1. It may thus be easily seen that a magnetic field produced by two windings 90° apart and supplied with currents 90° apart is essentially a rotating one, the N pole traveling in a clockwise direction in Fig. 22. Three windings spaced 120° apart and supplied with three phase currents 120° apart likewise produce a rotating magnetic field. The stator of an actual induction motor is not built exactly as shown; there are no separate projecting pole pieces; the different windings are imbedded in slots, like the winding of a D.C. armature.

The rotor, or moving part, consists of a laminated iron core accurately fitted to turn between the poles of the stator. In the periphery of the rotor are imbedded conductors which may be interconnected in different ways. In the squirrel cage winding the conductors are all short circuited on one another by being connected to conducting end rings. Or the rotor may be equipped with wound coils and the ends of the coils be connected to slip rings. Brushes bearing on these rings make it feasible to short circuit the coils if desired or else the brushes may be connected together through resistances, thereby increasing the resistance of the rotor circuit. This scheme of having a wound rotor and inserting external resistance when desired is used in most large size motors; the squirrel cage rotor is generally not used in motors over 15 H.P. in capacity. Motors built for special service, however, may have squirrel cage rotors in sizes as high as 75 H.P.

Consider the squirrel cage rotor in the rotating magnetic field. If the rotor is standing still and the magnetic field is revolving it is evident that an E.M.F. will be developed in the rotor winding and as the winding is short circuited, a current will flow in the rotor conductors. But it is a fundamental principle that a conductor carrying current placed in a magnetic field will be acted upon by a force. By consideration of the motion of the field, direction of induced E.M.F., etc., it may be shown that the force acting in the rotor conductors will be in such

a direction that the rotor is urged to revolve in the same direction as the magnetic field. Just so long as there is relative motion between the rotor and magnetic field, a torque will be produced which tends to accelerate the rotor. If there were no losses of any kind in the revolving rotor it would continue to accelerate until the relative motion of rotor and field was zero, i.e., the rotor would turn at the same angular speed as the field, called synchronous speed; the slip is then zero. There always exists some brush friction and windage to overcome, so that it is always necessary for the rotor to exert some torque, therefore the rotor never quite reaches synchronous speed; at no load the slip may be between .2% and 1.0%.

Now as the rotor is called upon to exert more torque, more current must flow in the rotor conductors; this can only occur if a greater E.M.F. is induced in them, which in turn requires an increase in the slip. The slip must therefore increase with load; for small loads the slip and load are nearly proportional.

The effect of increasing the rotor resistance is to increase the slip necessary to exert a certain torque. The reason for this is almost self evident. To exert a certain torque requires a certain current in the rotor; but if the resistance of the rotor circuit is increased, the E.M.F. must be correspondingly increased to produce the required current. The E.M.F. can only increase by an increase in slip. The increase in slip for a certain torque, by increase of the rotor circuit resistance, is only obtained by a decrease in efficiency. Whatever heat is generated in the external resistance added to the rotor circuit, is just so much loss, as it is useless in producing turning effort in the motor.

The maximum torque which an induction motor can exert is independent of the variations of the rotor resistance. But the speed at which this maximum torque occurs decreases with increase of resistance. Therefore the "pull out" point, or maximum output of the motor, decreases with addition of external resistance to the rotor circuit.

The power factor of the induction motor is very low at light loads, increases with load up to about rated load, and then decreases

again with overload. It may be about 30 per cent at no load and rise to between 80 and 93 per cent for maximum value, the higher figure being for large size motors.

At standstill the induction motor is essentially a short-circuited transformer, the rotor corresponding to the secondary of a transformer. But if a short-circuited transformer is connected to a line of normal voltage the current taken is excessive, being perhaps 20 or 30 times full load value. Because of magnetic leakage between rotor and stator the conditions in the induction motor at starting (rotor at standstill) are not quite so bad as

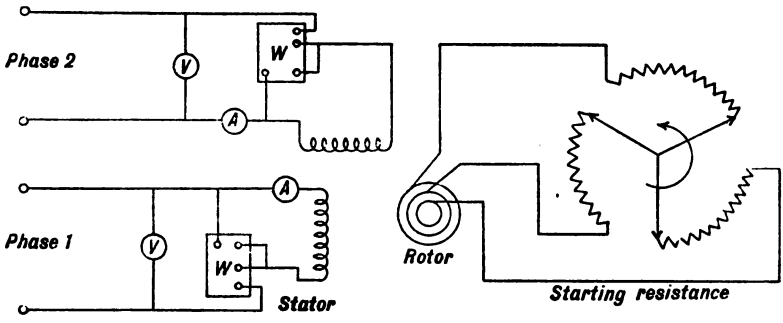

FIG. 23

with an ordinary transformer. However, the starting current when an induction motor is connected to a line of normal voltage is excessive and it is the principal object of the added external rotor resistances to limit this starting current.

Another effect of introducing resistance in the rotor circuit at starting is to increase the starting torque. An induction motor exerts its maximum torque when the rotor reactance is just equal to the rotor resistance. The rotor reactance varies directly with the slip and at standstill is much greater than the resistance in the ordinary rotor winding. By inserting extra resistance at starting, the resistance is brought equal to the reactance and maximum torque is exerted at starting, a very favorable condition when the motor has to start line shafting, etc. As the

motor speeds up the extra resistance is cut out in steps and if properly done, the equality of resistance and reactance is maintained as the motor speeds up; thus the rotor may be made to exert its maximum torque all the time during which it is accelerating.

With a polyphase induction motor three runs are to be made, one with the rotor short circuited and two with additional resistance in the rotor circuit. If a two phase motor is used make connections as in Fig. 23, and if three phase, as in Fig. 24. Note the effect on the direction of starting of reversing the connections of one phase; of reversing two phases.

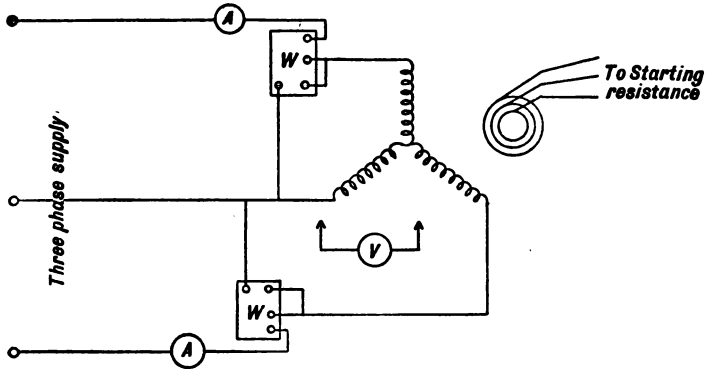

FIG. 24

Instead of using two sets of instruments as shown in above figures, a single set of instruments may be used with the combination of switches given in Ex. 4.

In measuring three-phase power with two wattmeters as above, one wattmeter will read negative if the power factor of the motor is less than .5. Begin the run at full load and connect the wattmeters so that positive readings are obtained on both phases; at full load the power factor will surely be greater than .5. As the load decreases one wattmeter reading will decrease faster than the other, will reach zero at perhaps one-quarter load and at lighter load will deflect backward. Under this condition the

potential coil is to be reversed, reading taken and *called negative*, i.e., the motor input is the difference of the two wattmeter readings. No such difficulty will be encountered with the two-phase four-wire systems given in Fig. 23, but may arise if a three-wire two-phase system is used for power.

Calculate the full load torque of the motor (assuming no slip) and take readings of input, torque and speed for about eight values of torque between zero and 50 per cent overload. Keep impressed voltage and frequency at rated values of the motor.

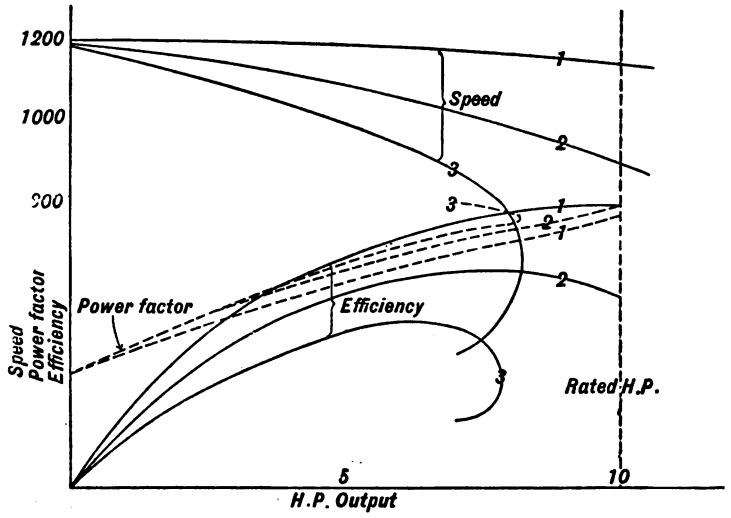

FIG. 25

Make this run with the rotor short circuited and make similar runs with two different values of added resistance in the rotor circuit. The greater value of resistance should be about sufficient to give the rated full load torque at half synchronous speed; the other resistance to be about one-half this value.

The curves obtained from test should look similar to those of Fig. 25, which were obtained from a 10 H.P., 60 cycle, 220 volt, 2 phase motor.

The next characteristic of the motor to be investigated is the relation of starting torque and current, to the resistance of the rotor circuit. Apply one-half rated voltage (at rated frequency) to the motor, having the rotor locked. Take readings of torque and current for the different points of the starting resistance.

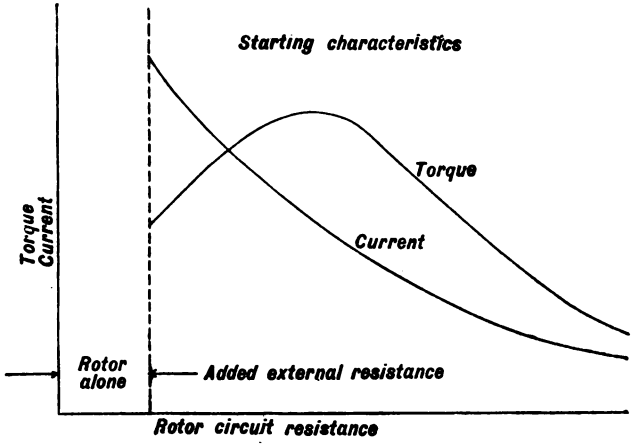

FIG. 26

Measure resistance of rotor and of the various steps of the starting resistance, if they are not given. Plot curves of starting torque and current against rotor circuit resistance. The results should be somewhat of the form of those given in Fig. 26, which are for the same motor as the curves of Fig. 25.

EXPERIMENT VI

The Synchronous Motor; Phase Characteristic and Phase Shifting with Load. If two alternating current generators are operating in parallel, supplying power to the same line, and the driving power is taken away from one of them, it will ordinarily continue to run at exactly the same speed it had before (provided the other alternator does not slow down) and it draws from the other alternator the power necessary to run itself. An alternator so running is termed a synchronous motor as it runs exactly in synchronism with the alternator supplying its power. This point is to be emphasized; a 6-pole synchronous motor running from a 60-cycle power line runs at 1200 r.p.m. no load and 1200 r.p.m. full load, and continues to run at 1200 r.p.m. as the load is further increased, until the " pull out " point of the motor is reached at perhaps 50 per cent overload; when this load is reached the motor pulls out of synchronism with the line and stops. As soon as it pulls out of step it draws an excessive current and the protective apparatus either at the motor or in the generating station must open if disastrous results are not to be incurred.

A synchronous motor is generally not self-starting; some auxiliary driving power must bring it up to synchronous speed and when the proper conditions are reached the switch connecting the motor to the supply line is closed. A polyphase synchronous motor is sometimes started as an induction motor in which case of course, no auxiliary starter is necessary. In this method of starting the field is left unexcited, the armature is connected to some low voltage taps (generally 50 per cent normal or less) on the supply transformers. The armature draws a rather heavy current, perhaps 100 per cent in excess of full load, and this armature current induces eddy currents in the pole faces. The interaction of the armature current and these eddy currents

tend to make the armature rotate. It will continue to accelerate until it reaches synchronous speed; the armature is then connected to the normal voltage taps of the supply transformers and the field gradually excited until normal field current is reached. In revolving field synchronous motors where this method of starting is used, each pole face of the machine is pierced by a number of brass or copper rods placed parallel to the shaft; the ends of these rods are connected to brass short circuiting rings. The machine thus starts as a squirrel cage induction motor.

The connection of the armature to one-half voltage taps and then to normal voltage taps is accomplished by a double throw switch; the transition is made as quickly as possible so that the motor has no time to slow down during the process.

The disadvantage of this method of starting is the large starting current taken from the line at very low power factor; the resultant fluctuations in the line voltage may seriously interfere with the operation of other synchronous apparatus connected to the line; in fact, if the fluctuations in line voltage are large, other synchronous apparatus may actually fall out of step. In spite of these disavantages the method is a very common one for starting revolving field synchronous motors.

Another method for starting a synchronous motor is to use a small induction motor, the rotor of which is mounted on the extended shaft of the synchronous motor. The induction motor may be 10 per cent the size of the synchronous motor; it must have at least one pair of poles less than the synchronous motor; and it is so designed that when its load is equal to the no load losses of the synchronous motor (core loss and friction) it is turning the synchronous motor at synchronous speed. After the synchronous motor is connected to the line, power is cut off from the induction motor and the rotor runs idle.

The switch connecting the armature of the synchronous motor to the supply line is called the synchronizing switch. Before this may be closed four conditions must be fulfilled, unless the induction motor principle, as described above, is used for starting the motor. These conditions are:

1. Motor voltage equal to line voltage.
2. Motor frequency equal to line frequency.
3. Phase of motor voltage exactly opposite to that of line.
4. Wave form of motor E.M.F. must be nearly similar in form to wave of line E.M.F.

The first three conditions may be adjusted by the operator, the fourth is satisfied or not when the machine is built. As was mentioned in the discussion of the alternator, the wave form of an A.C. machine is determined by the shape of air gap and pole piece and the distribution of the armature windings.

By analyzing the four conditions named, it is seen that they demand that the motor voltage shall at every instant be equal and opposite to the line voltage. If that were not the case a

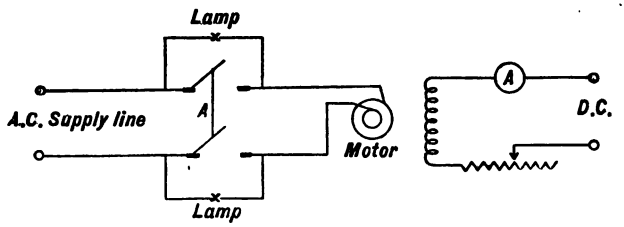

FIG. 27.

heavy current would flow through the armature due to the unbalanced voltage; the current will be large even for a small unbalancing because of the low impedance of the armature.

The first two conditions can be satisfied by the use of a voltmeter and speed counter when frequency of line supply is known. To determine the third condition, synchronizing lamps or a synchronoscope must be used. The synchronoscope is an indicating instrument having two internal circuits, one of which is connected to the line and one to the motor. The position of the pointer indicates the relative phases of the two E.M.F.'s.

Lamps are to be used in this test, so their action will be more fully described. One lamp is connected across each blade of the synchronizing switch *A*, as in Fig. 27. The lamps will flicker as the motor phase changes with respect to the line phase. They complete a circuit consisting of the motor armature, the

line and armature of the generator supplying the line. In this circuit there are acting two E.M.F.'s, that of the motor and that of the generator. At any instant the effective voltage in causing current to flow through the lamps, is the resultant of the two; as their relative phase changes, the resultant changes and has for its locus a circle as shown in Fig. 28. The resultant voltage for any position of E_m is seen to be OR, the chord to the circle from the point O. The current through the lamps, and therefore their brilliancy, will increase as the motor voltage E_m catches up (in phase) with E_g and will decrease as the two E.M.F.'s separate in phase. There will be no current through the lamps when E_m

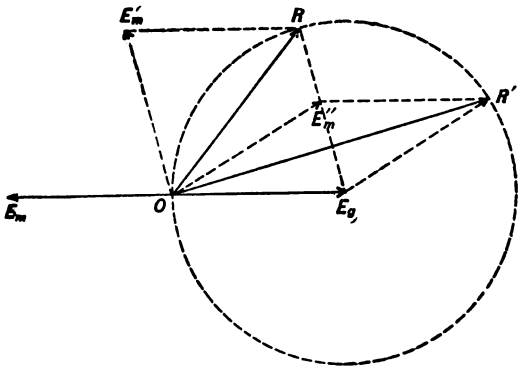

FIG. 28

is just opposite in phase to E_g; condition No. 3 is therefore satisfied when the lamps are dark and this is called the "*dark*" connection for synchronizing. With the lamps connected diagonally across the switch, the proper time for closing the switch is at the middle of a *bright* period.

After having adjusted the motor voltage to equal the line voltage, the speed of the motor is so adjusted (by adjusting its driving motor) that the lamps flicker once in 4 or 5 seconds; the switch A is then closed in the middle of a dark or bright period, according to the connection of the lamps. The power supplying the starting motor may then be cut off.

If a three phase motor is used, it is necessary to use three synchronizing lamps, one connected across each pole of the three pole synchronizing switch. The lights must flicker together; if they are brilliant and dark in rotation instead of simultaneously, one of the phases is incorrectly connected and two of the supply lines must be reversed in their connection to the motor.

One of the important features of the synchronous motor is well shown by the set of curves called " phase characteristics."

If the load on a synchronous motor is kept constant and the

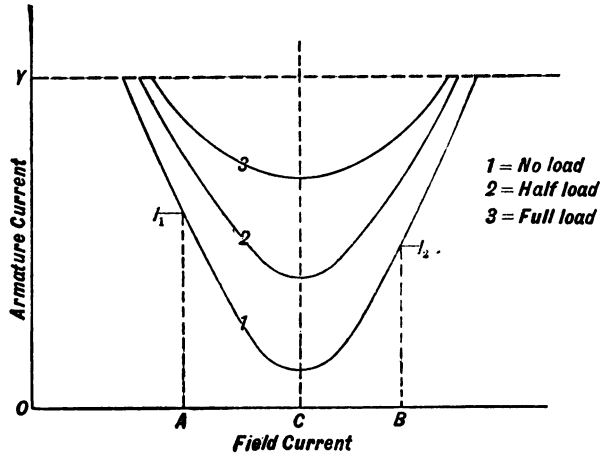

FIG. 29

field current varied, the armature current is caused to vary in about the fashion indicated in Fig. 29. It is this curve, drawn between armature current and field current as variables, which is called the phase characteristic. For any load a certain field current gives a minimum armature current. For any other value of field current the armature current is greater. Now as the load on the motor is supposed constant the power component of the armature current must remain practically constant. It must, therefore, be that any other than normal excitation on the motor produces in the armature a wattless current. In fact an

overexcited synchronous motor draws a leading current from the supply line and an underexcited motor a lagging current, as shown in Fig. 30. OE is the phase of the line voltage and OI is the armature current for no load with excitation OC, Fig. 29. Excitations OA and OB result in armature currents AI, and BI_2, Fig. 29, which are represented vectorially in Fig. 30, as OI_1 and OI_2.

If the speed of a synchronous motor is independent of load

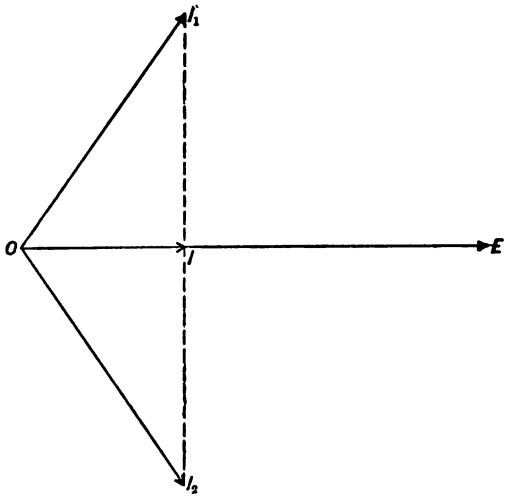

FIG. 30

the question arises, how does the motor adjust itself to take more or less load? The phase shifting of the motor armature is what accomplishes this end.

The E.M.F. causing current to flow through the armature of the motor, is the vector resultant of the motor voltage and line voltage. This resultant varies with the variation of the phase difference of the two E.M.F.'s. In Fig. 31 are shown two positions of E_m, the motor voltage, with respect to a fixed line voltage E_l. The resultant, OE_r, is the E.M.F. causing current to flow in the motor and this changes widely with a small shifting

of OE_m. The current OI is equal to the voltage OE_r divided by the armature impedance. It is laid off behind OE_r by the angle θ, where

$$\tan \theta = \frac{\text{armature reactance}}{\text{armature resistance}}.$$

It is variation of the angle α, giving the phase position of the armature, which permits the motor to take more or less load. The variation in α may be easily determined. Suppose that two insulated discs with metallic strips (as used in Ex. 1) are placed one on the shaft of the generator supplying the power and one on the shaft of the motor, and that an incandescent lamp

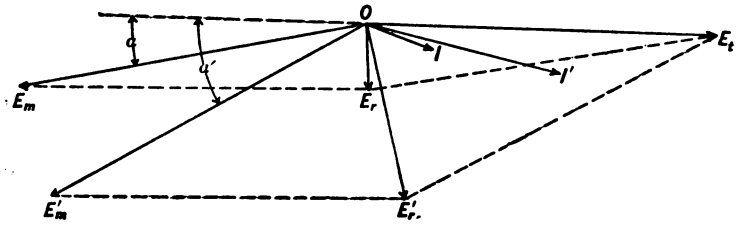

FIG. 31

is wired to a 110-volt circuit through these two discs and their brushes as in Fig. 32. The lamp will light up only if the two brushes are in contact with the metal strips at the *same time*. The brush on the motor moves over a graduated arc so that its position may be read and the brush on the generator remains stationary. The position to which the motor brush must be moved before the lamp is lighted gives the phase position of the motor armature with respect to the generator armature. If the generator is large compared to the motor, its armature position is also a measure of the phase of the E.M.F. impressed on the motor. It will be found that the value of α is almost directly proportional to the load.

With the connections as in Fig. 33, obtain the phase characteristics of the motor for no load, $\frac{1}{2}$ load and full load. On each curve get about 8 points, using for armature current not more

than 150% of the rated current of the motor; i.e., in Fig. 29, OY should not be greater than 50% above the rating of the motor. The motor may be loaded by Prony brake or, what is more convenient, by a D.C. generator. The generator may be used as a starting

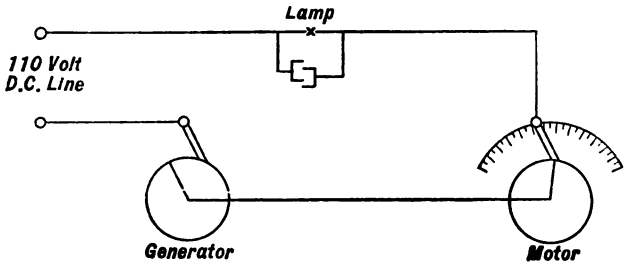

FIG. 32

motor until the A.C. motor is synchronized and then it may be used as generator to load the synchronous motor.

In getting the phase characteristics it is not necessary to know the exact load on the motor; ½ rated current may be used as half load, etc.

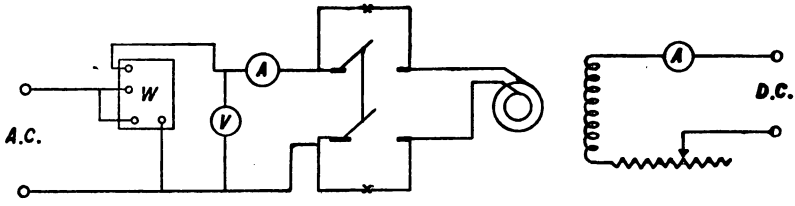

FIG. 33

The impressed voltage and frequency are to be held constant; read volts, amperes, watts input and field current.

Then adjust the field current to the value for minimum armature current at no load. Adjust the brush on the motor disc until the lamp burns and read brush position. If a low voltage lamp is not available, a D.C. voltmeter may be used instead of the lamp; the movable brush is shifted until the volt-

meter reads a maximum. Read also impressed E.M.F. and field current, which are to be maintained constant, and also watts and amperes supplied to armature.

Put on load in about 8 steps from no load to 50 per cent overload, taking same readings as for no load.

Plot on one sheet, against field current as abscissa, watts input, amperes input and power factor for each of the runs.

Plot on a second curve sheet, watts input to motor (armature input only) against brush position as abscissa. Note whether the brush is moved with or against the direction of rotation as load is increased, and so determine whether the motor armature is advanced or retarded in phase as the load on the motor is increased.

EXPERIMENT VII

The Rotary Converter; Effect of Voltage and Speed upon Ratio; Operating Characteristics. The efficient transmission of electric power over any considerable distance requires the use of alternating currents, as was explained in Ex. 4. Practically all motors for driving eletcric trains are direct current motors; although alternating current motors have been used in some few cases, the field of electric traction in America is so exclusively controlled by the direct current motor that from the general standpoint the A.C. motor need not be considered. The characteristics of a small A.C. series motor are given in Fig. 34. Also in dotted lines are given the characteristics of the same motor run with direct current power; the superiority of its behavior as a D.C. motor is so evident that nothing further need be said regarding the comparative merits of A.C. and D.C. railway motors.

The electric railway generating station is always located where water and coaling facilities are good, and A.C. power is generated. This power is transmitted at a high voltage to a substation where the voltage is stepped down. It is the function of the rotary converter to change this low voltage A.C. power into D.C. power for use in the car motors.

An elementary explanation of the performance of a rotary converter may be given by supposing that the winding of a D.C. shunt motor is tapped (on the end opposite to the commutator) at two points 180° (electrical) apart. These two taps are to be connected to two slip rings. Now any winding revolving in a magnetic field generates an alternating E.M.F. If the winding is connected to slip rings an alternating current is delivered to

the outside circuit; if the winding is connected to a commutator the delivered current will be direct. From this it follows that the shunt wound motor would have on its slip rings an alternating E.M.F.; if the rings are connected to an external circuit an alternating current will flow from the A.C. brushes and a corresponding increase in the D.C. input will occur. Such a machine converts D.C. power into A.C. power; it is ordinarily termed an

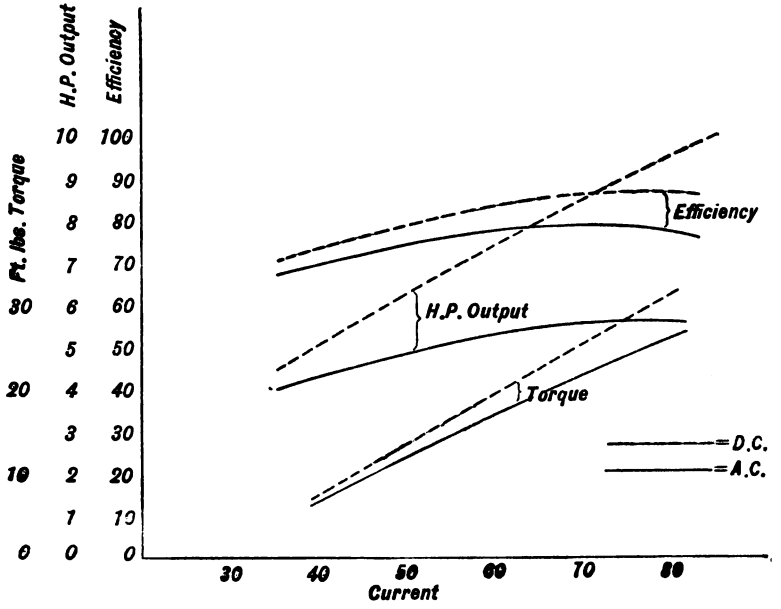

FIG. 34

" inverted " rotary. If now the function of the machine is reversed, i.e., A. C. power is supplied to the slip rings, the machine running as a synchronous motor, and D.C. power is taken from the commutator end, it will be operating as a rotary converter normally does. The amount of A.C. power input depends upon the amount of D.C. power output.

The efficiency of a rotary varies, of course, with the size, being perhaps 95 per cent in the larger machines and 85–90 per cent in the smaller ones.

The distinctive feature of a rotary of the ordinary type is the ratio of A.C. to D.C. volts. For a single phase machine it is .707, for a three-phase machine .612; it is a constant which depends only on the number of A.C. taps. For a given impressed A.C. voltage the D.C. voltage is nearly independent of load; it decreases somewhat with increase of load because of the impedance drop in the armature windings.

The power factor of the A.C. input will depend upon the value of the field current; in this respect the rotary is exactly similar to the synchronous motor. Under ordinary conditions

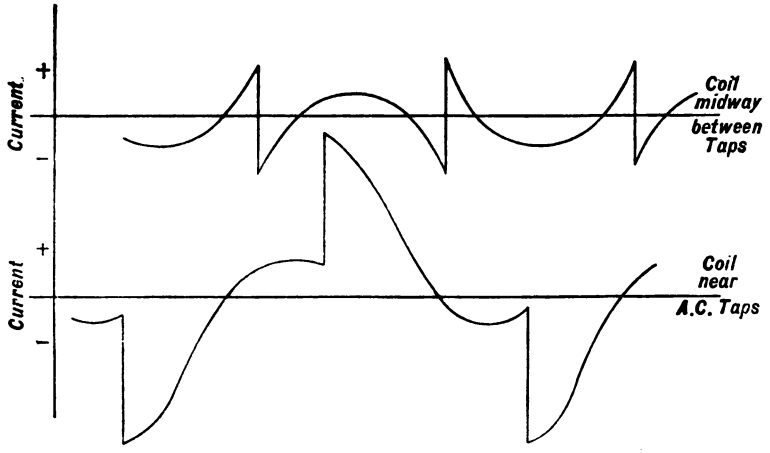

FIG. 35

the field current should be adjusted to that value which gives a minimum armature current for the load being carried; this value of field current does not change much with the load; if adjusted for one load it will be nearly correct for any other.

The coils of the armature carry the instantaneous difference of the A.C. and D.C. currents, the form of the current flowing in any one coil being displaced sections of a sine curve. The form of the current wave in the coils, one close to the tap and one half way between taps is shown in Fig. 35. The peculiar shape of these current waves results in unequal heating in the different coils of a rotary, those nearest the taps getting hottest.

Because of the fact that the D.C. and A.C. currents in the different coils tend to neutralize one another the capacity of a given machine is considerably greater run as a rotary than as an alternating or direct current generator. A certain size machine used as generator or rotary has capacities as follows:

D.C. generator capacity	= 100 K.W.
Single phase A.C. generator capacity	= 70.7 K.W.
Single phase rotary capacity	= 84.8 K.W.;
Three phase rotary capacity	= 133.8 K.W.;
Six phase rotary capacity	= 193.7 K.W.

The above values are for power factor of one. For other power factors the values are somewhat different.

Because of the fact that the ratio of D.C. to A.C. voltage is fixed for any given rotary, it might seem that a rotary could not be compounded, i.e., give a D.C. voltage increasing with load. In one way this is true; for a given impressed A.C. voltage the D.C. voltage cannot be increased, but in practice the impressed A.C. voltage is made to increase with load, thereby making the D.C. voltage rise also.

Recently a special type of rotary using field poles in two sections has been developed; in this rotary the ratio of E.M.F.'s is not fixed. In a small laboratory machine of this type, by proper adjustment of the currents in the two sections of the pole, the D.C. voltage may be made to vary from 90 to 135, while the impressed A.C. voltage is kept constant at 88. This is a very special type of machine and not much used as yet.

The ratio of an ordinary rotary having the A.C. taps separated by the angle α, measured in electrical degrees, is given by the formula,

$$\text{A.C. volts} = \frac{\text{D.C. volts}}{\sqrt{2}} \times \sin \frac{\alpha}{2}.$$

There are several methods for starting a rotary and bringing it into synchronism with the A.C. line. Either of the two methods described for a synchronous motor may be used and there is in

the rotary the additional possibility of starting from the D.C. end as a D.C. shunt motor. This is a convenient method for use in the laboratory as D.C. power is always available. In substations D.C. power may not be available, under which condition one of the methods employing A.C. power must be used.

For starting from the D.C. end it is necessary to have a D.C. line of at least the rated voltage of the machine. For a single phase machine rated at 110 volts A.C. the necessary D.C. voltage is $110/.707 = 157$ volts; for a three-phase 110 volt A.C. rotary, the necessary D.C. voltage is $110/.612 = 181$ volts. A D.C.

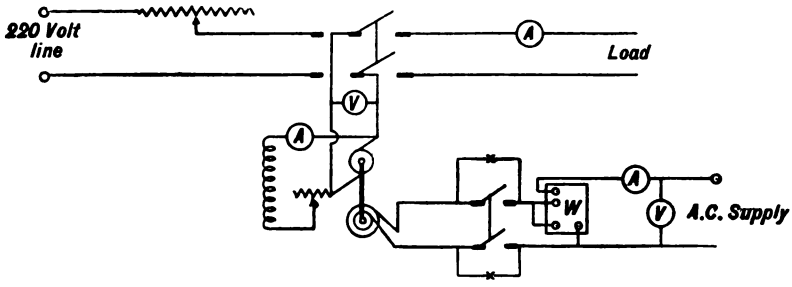

FIG. 36

starting rheostat is used as with a D.C. motor. The rheostat resistance is decreased until the impressed D.C. voltage is just that necessary to give the required A.C. voltage. Then the rotary is bought into synchronism with the line by changing the field current; a synchroscope or synchronizing lamps, as with the synchronous motor, may be used to indicate the proper time for closing the synchronizing switch.

Two runs are to be made with the rotary, one running from the D.C. end to determine the ratio of the converter and its possible variation and the second to get the operating characteristics of the machine when running from the A.C. end, as it is normally designed to do.

For making these two runs it is convenient to make connections as given in Fig. 36. The rotary represented is a single-phase machine designed for 110 volts on the A.C. end.

In testing the voltage ratio make one run, keeping the impressed D.C. voltage constant, and vary the speed through as wide a range as possible by changing the field strength; read A.C. volts, D.C. volts and speed. Then take another run with various D.C. voltages impressed, keeping the speed constant by variation of field strength; read same as before.

In taking the load run the machine is first to be synchronized with the A.C. line. After the starting rheostat, *R* has been so adjusted that the rotary A.C. voltage is equal to that of the A.C. line, change the speed (by field variations) until the synchronizing lamps indicate synchronism, and then close the synchronizing switch in the middle of a dark period, with lamps connected as in Fig. 36. Of course "bright" connections of lamps may be used if desired. When synchronized the D.C. supply line is to be opened and the switch thrown over to load side.

Adjust the value of field strength so that at no load the A.C. armature current is a minimum. Leave the field circuit resistance constant at this value and put load on the D.C. end of the rotary in about 8 steps between zero and 50 per cent overload. Keep impressed A.C. voltage constant. Read A.C. volts, amperes, and watts, field current, D.C. load current and D.C. volts.

On one curve sheet plot the ratio values obtained in first run, using ratio as ordinates. On second curve sheet, plot curves of D.C. volts, efficiency and power factor, using D.C. load current as abscissæ for all curves.

EXPERIMENT VIII

Parallel Operation of Alternators; Circulating Current; Division of Load Dependent upon Phase Shifting. At present the largest sizes in which it is feasible to build A.C. generators is about 20,000 K.V.A. To equip a station for a capacity of 100,000 K.V.A. it is therefore necessary to install several generators, and as there is generally only one set of bus bars and one distribution

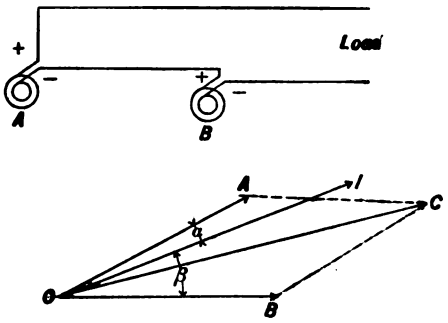

FIG. 37

system, it is necessary to so connect these generators electrically, that they all supply power to the same line. The only stable connection is to have them working in parallel, and many stations have as many as ten or more large alternators all connected in parallel to the same switchboard. It is therefore important to investigate the operating characteristics of such a set of machines.

It will first be shown that two alternators operating in series are not in stable equilibrium. Two single phase machines, connected in series, supplying a load drawing a current, lagging

somewhat behind the line E.M.F., are shown in Fig. 37. Each machine is generating the same voltage and it is supposed that for some reason that machine B has pulled slightly ahead of machine A in phase. The vector diagram of E.M.F.'s and current is given in Fig. 37. The vectors OA and OB represent the machine voltages, OC the resultant or line voltage, and OI the line current. The load on machine B is equal to $OB \times OI \times \cos \beta$ and that on machine A is equal to $OA \times OI \times \cos \alpha$. As α is less than β it is evident that machine B, which for some reason has pulled ahead of A in phase, has thereby relieved itself of part of its share of the load. This effect will make machine

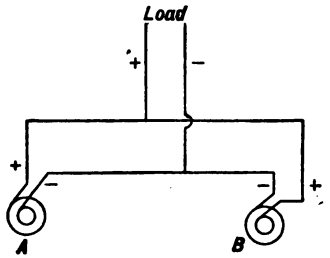

FIG. 38

B speed up still more because the prime mover will increase its speed if load is taken from it. B will continue to get ahead of A in phase until the two vectors OA and OB are practically in opposition, under which condition there is no line voltage and therefore no load.

The polarities marked in Fig. 37, of course, are true only at a certain instant, but the two machines will both reverse at the same time, leaving the phase of E.M.F.'s, relative to one another, the same. If, however, when the two machines have pulled into opposition, with respect to each other, the load circuit is short circuited and the load attached as in Fig. 38, the conditions of operation and load distribution are stable. The vector diagram given in Fig. 39 will make this point clear. When the two machines are exactly in opposition, with respect to each other,

the E.M.F. vectors are shown at OA and OB. There is no resultant E.M.F. in the circuit consisting of the two armatures and therefore there is no current flowing around this local circuit. The two machines will divide the load equally provided that the two armatures have equal impedances.

In Fig. 38 it is seen that although the two E.M.F.'s oppose one another around the local armature circuit, they act together, in parallel, in so far as the load circuit is concerned.

Suppose now that machine B speeds up for an instant with respect to A, so that the relative phases of the two E.M.F. vectors are as shown as OA and OB', in Fig. 39. In the local circuit there now exists a resultant voltage OC which will force current to flow through the two armatures, *in addition to whatever load*

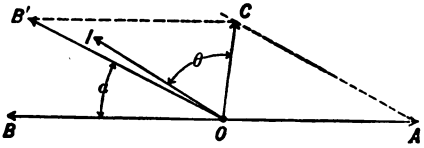

FIG. 39

current the two machines may be carrying. As the inductance of the armature is much greater than the resistance, this local current will lag nearly 90° behind the E.M.F. causing it; OI represents this current in phase, behind the voltage OC, by the angle θ where

$$\tan \theta = \frac{\text{armature inductance}}{\text{armature resistance}}.$$

Now this current is nearly in phase with the voltage OB', and hence is a load on machine B, while it is a motor current for machine A, as it is nearly 180° out of phase with OA. The real effect of this current is therefore to take some of the load from machine A and put it on machine B; this will tend to slow down B and is therefore an effect which tends to prevent the machines leaving their phase position of 180° with respect to one another.

The previous analysis has been on the assumption that the

two machines were generating the same voltage. Suppose now that the load on the two machines is equally divided (E.M.F.'s, 180° apart) and the excitation of machine *A* is increased. Will this change the distribution of load? By reference to Fig. 40, it is seen that the resultant voltage *OC*, now lies in phase with *OA*. The resultant local current *OI* is shown lagging by the angle θ behind *OC*; this current *OI* is practically a wattless current, as it is in 90° position, nearly, with respect to both E.M.F.'s. As it is not in phase with the generator E.M.F.'s, it cannot, with non-inductive load, represent load current. As a matter of fact, this increase in voltage of machine *A* will scarcely affect the load distribution at all, but will produce a current which flows

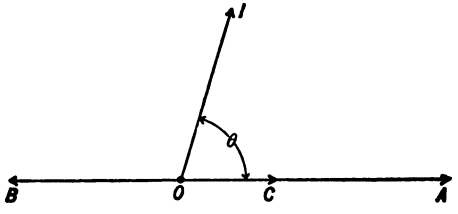

FIG. 40

in the local circuit only; it is practically 90° out of phase with the E.M.F.'s and its only effect is to tend to equalize the two voltages *OA* and *OB*. It will tend to magnetize the machine *B* and demagnetize machine *A*. This effect of armature reaction by the circulating current will change the voltage of the line somewhat as excitation of machine *B* is varied.

Referring to Fig. 39, it is evident that the division of load depends upon the angle α. Now the only way of changing this angle is to vary the driving torque of the prime mover, and it is in this fashion that load is distributed between the different machines in a station. The steam supply of the driving engine or turbine is generally under the control of the switchboard operator.

Summing up the conclusions reached we have: the division of load between two alternators operating in parallel depends only upon their relative phase position; the load division cannot be effected by varying the field strength, but such variation of field results in a wattless current circulating in the local circuit, which merely heats the machines, represents no power output and is therefore detrimental to the operation of the machines.

The first run to be made in this test is to show the variation of load on machine B by variation of the phase angle α, with constant excitation of B; the second run is to keep α fixed and to vary the excitation of B both above and below normal to show variation of circulating current and independence of load.

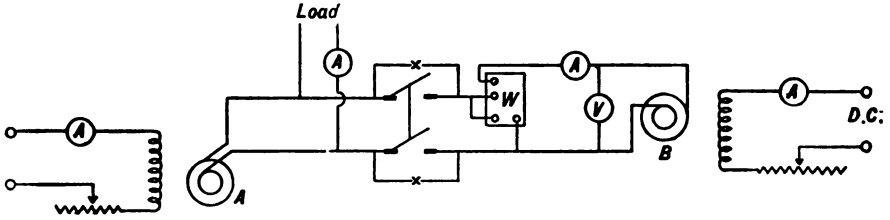

FIG. 41

Make connections as in Fig. 41, bring A to rated voltage and read its field current. Keep it constant at this value throughout the test. Put upon machine A, a load of about one-half its rated capacity. Synchronize machine B with the line and read field current, watts output, armature current and line voltage. Leave the field current fixed at this value and increase the torque of B's prime mover in such steps as will produce changes in the armature current of B of about one-quarter rating. Take readings up to 50 per cent overload, reading for each step watts and armature amperes of B and line voltage. Get the values of α for each setting of load by the scheme of two insulated discs used in experiment No. 6.

Then reduce the torque of B's prime mover until the wattmeter reads zero. Now vary the field current of B in such steps as

produce increments in the armature current of about one-quarter rating, reading for each setting, field current, armature current and watts, line voltage and phase position of *B*'s armature.

On one curve sheet plot the variations of watts load of *B* and phase position of its armature. Calculate also the value of the circulating or wattless current flowing in *B*'s armature for each reading. Plot load and circulating current against phase position as abscissæ. On a second sheet plot variations of load in watts, circulating current and phase position, against field current of *B* as abscissæ.

NOTE.—The remarks regarding effect on load distribution of variation of *B*'s field current were made on the assumption that the angle θ was nearly 90°. In so far as this is not true the conclusions reached are more or less inaccurate, but a more detailed discussion makes the question too complex.

EXPERIMENT IX

Current and Voltage Relations in a Three Phase Circuit; Measurement of Power on Non-Inductive and Inductive Loads. Practically all A.C. power is generated, transmitted and utilized by polyphase circuits and machines. Of all polyphase circuits the three phase is by far the most important.

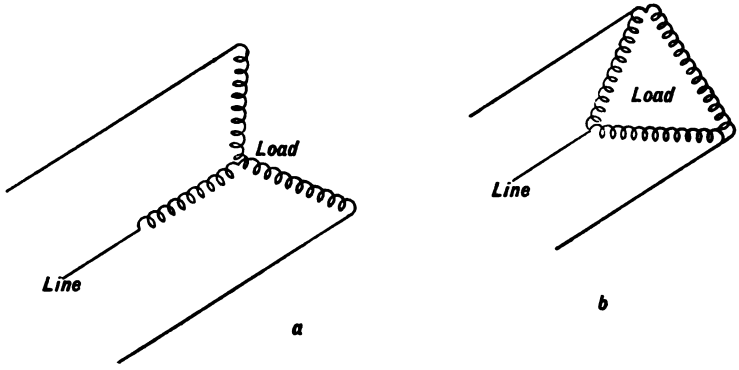

FIG. 42

The easiest way to study the current and E.M.F. relations in such a circuit is by first considering it as three single phase circuits. The problem will be investigated only for balanced loads, i.e., a polyphase system which may be considered as made up of a group of equally loaded single phase circuits.

A three phase load may be connected in two ways: the star or *Y* connection shown at *a*, Fig. 42, and the mesh or Δ shown at *b*. With either connection only three wires are used. Apparently only three wires are needed for the Δ connection, and it may be shown that in the *Y* connection a wire connecting to the

center or neutral point of the load is unnecessary. In Fig. 43 the three phases are supposed separate, each phase is carrying the same magnitude current and of course the three currents, represented as vectors, are 120° apart. This feature of the three phase systems is a result of the method of placing the coils on the armature of the three phase generator; the coils are placed 120° (electrical) apart, hence generate three sine E.M.F.'s 120° apart, and so the currents from such a generator are 120° apart.

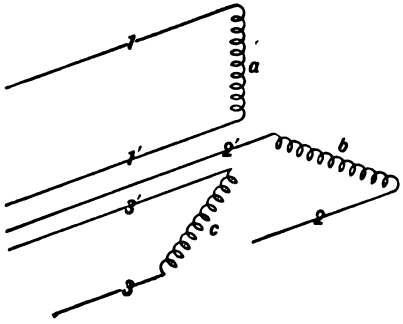

FIG. 43

In Fig. 43 the three single phases are shown at *a*, *b*, and *c*. The three lines, 1′, 2′ and 3′ are evidently carrying three equal currents, 120° apart in time. If then these three lines are joined throughout their entire length, the resultant single line will carry the resultant of three equal sine currents spaced 120° apart. But such resultant is zero, and therefore the combination line or neutral, as it is called, is useless and so not used. The three single circuits, joined together at 1′,2,′3′ then constitute a *Y* load, supplied with three phase power through the lines 1, 2, 3. In discussing voltage and current relations we have,

i = current in each phase of load;
e = voltage across one phase;
I = current in any line;
E = voltage between lines.

In the Y connection of load it is evident that $I=i$. To get the line voltage it is necessary to take the vector difference of two voltages, each of magnitude $=e$, spaced 120° apart. The voltage between lines by this construction gives $E=e\sqrt{3}$.

In the delta connection $E=e$. To get I it is necessary to take the vector difference of two currents each of magnitude i, and 120° apart in phase, and this gives $I=i\sqrt{3}$.

Now no matter how the load is connected, it is evident that the power used in the three phases is equal to $3ei$, if $\cos\phi=1$. Substituting values of line current and line voltage gives power of a three-phase load $=EI\sqrt{3}$, and this holds good for either con-

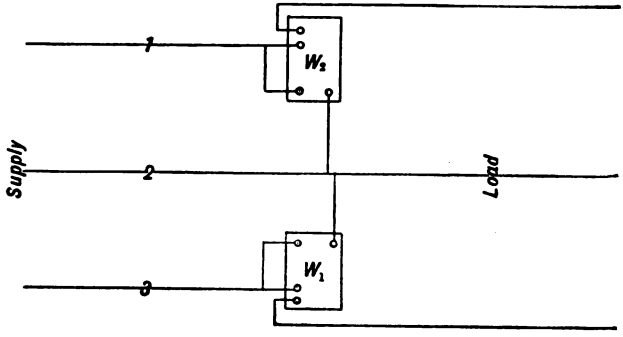

FIG. 44

nection of load. If the power in each phase is equal to $ei\cos\phi$, then in terms of line quantities we have watts used in three-phase load $=EI\sqrt{3}\cos\phi$.

To measure the power used in a three phase line it is not necessary to actually measure the watts in each phase and multiply by three. If it is known that the load is non-inductive, then the power used is easily determined by measuring the line current and voltage and multiplying by $\sqrt{3}$. When the power factor is unknown, as is generally the case, another method must be used.

If two wattmeters are used as shown in Fig. 44, the sum of the two readings will always be the power used in the three phase circuits, no matter what the power factor may be or whether the

load is balanced. If the power factor is less than .5, one wattmeter will indicate negatively and then the algebraic sum must be used, not the arithmetical sum. For balanced loads it may be shown that the current in line 1 is 30° out of phase with the voltage between 1–2, and the current in line 3 is 30° out of phase with the voltage between lines 3–2. If ϕ is the phase angle of the load,

$$W_1 = EI \cos(\phi + 30°);$$
$$W_2 = EI \cos(\phi - 30°);$$

from which

$$W_1 + W_2 = EI\sqrt{3} \cos \phi.$$

But we had already shown that this is the power used in the three phase load.

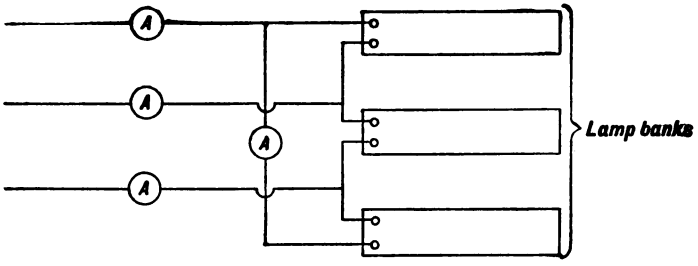

FIG. 45

With connections as in Fig. 45, test the accuracy of the above formula, for the values of line and phase currents in Δ connection; try both balanced and unbalanced loads.

Then using Y connection with inductive and with non-inductive loads, test the accuracy of the above formulæ for voltages and currents and also for three phase power with balanced and unbalanced loads. The three phase power may be measured first with the wattmeters connected in the three phase line as shown in Fig. 44; then the potential coils may be so connected that the wattmeters read the power in two of the phases directly by connecting those potential leads which, in Fig. 44, are connected to the center line, to the center of the Y on the load. When the test is made for unbalanced load, suppose that phases 3 and 2 (Fig. 46) are left balanced and the load on phase 1 is

decreased. Then with the potential coils connected to the outside wires and the center of the Y the total power used in the the three phase circuit will evidently be equal to (2 × reading of W_2 + reading of W_1). Then connect the wattmeters as in Fig. 44 and take readings of W_1 and W_2. Their sum should give the three phase power as measured above.

An easy way of balancing a Y-connected load is to open one phase and adjust the two remaining ones until the voltage drop across the two is equal. Then put in the third phase and open one of those already tested. Adjust the third phase to give a drop equal to the one left connected in series with it. Then all phases may be closed and the load will be balanced.

The connections for the tests on the Y load are given in Fig. 46. For both inductive and non-inductive loads, prove that $E = e\sqrt{3}$ and that $W_1 + W_2 = 3 \times$ (watts per phase). By ammeter,

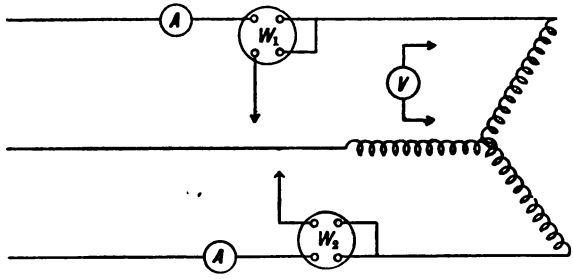

FIG. 46

voltmeter and wattmeter get cos ϕ of the load, testing one phase only (the load has already been balanced). See if it checks with the value as given by

$$\cos \phi = \frac{W_1 + W_2}{EI\sqrt{3}}.$$